Modeling, Design and Optimization of On-Chip Inductors and Transformers

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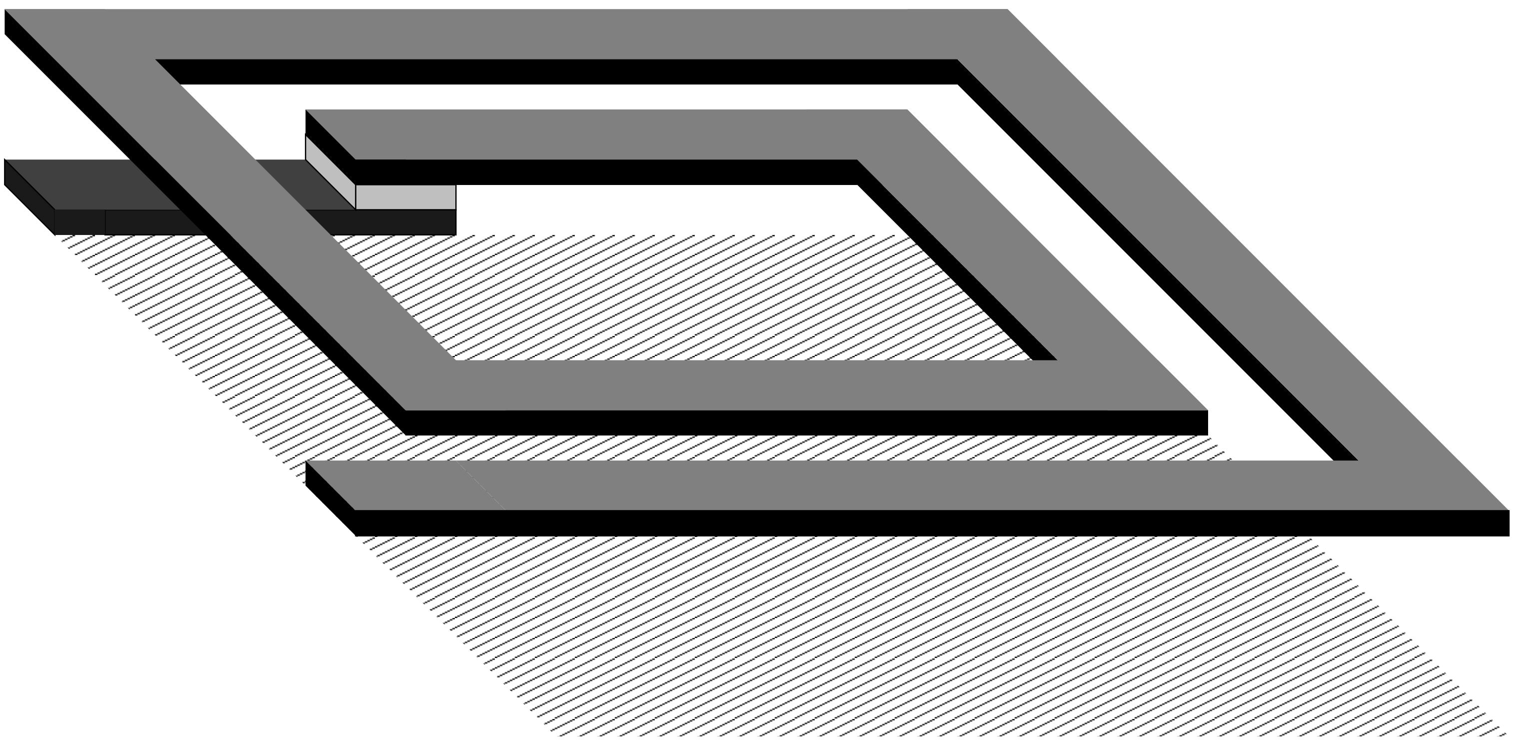
The Goal



Simple, Accurate Expressions for Inductance

OUTLINE

- Background
- Current Sheet Approach
- Accurate Inductance Expressions
- Optimization of Inductor Circuits
- Transformer Modeling
- Contributions

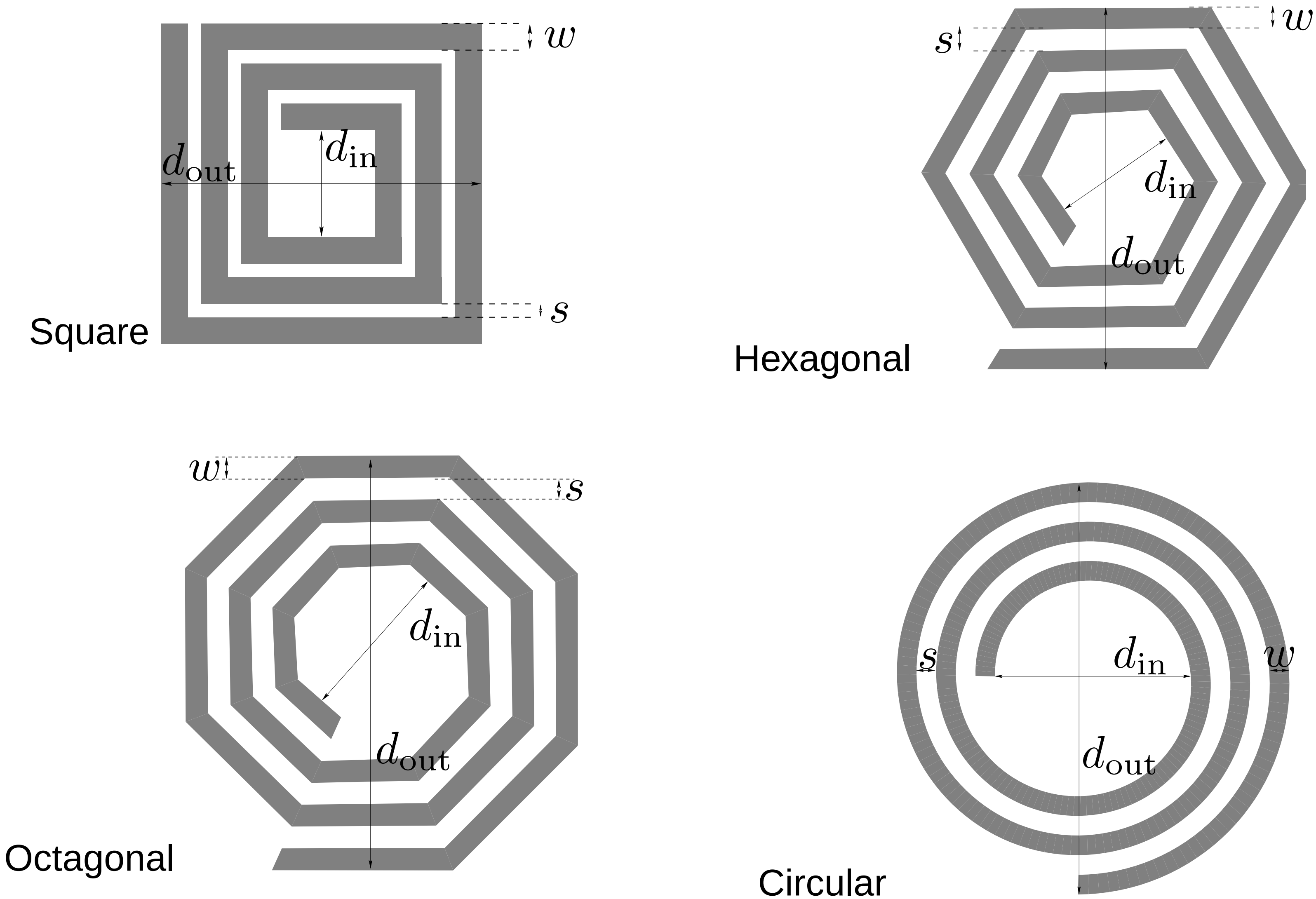
On-Chip Inductors and Transformers

- Essential for radio frequency integrated circuits (RFICs)
- Narrowband circuits
  - Low noise amplifiers, oscillators, filters, matching networks, baluns
- Broadband circuits
  - Shunt-peaking to enhance bandwidth

On-Chip Inductor Options

Attribute	**Bond wire**	**Planar Spiral**
Inductance	$0.5 - 4\text{nH}$	$0.2 - 100\text{nH}$
Q	$30 - 60$	< 10
Parasitics	C_{Bondpad}	R_{s}, C_{ox}, C_{si}, R_{si}
Fluctuations	Large	Small

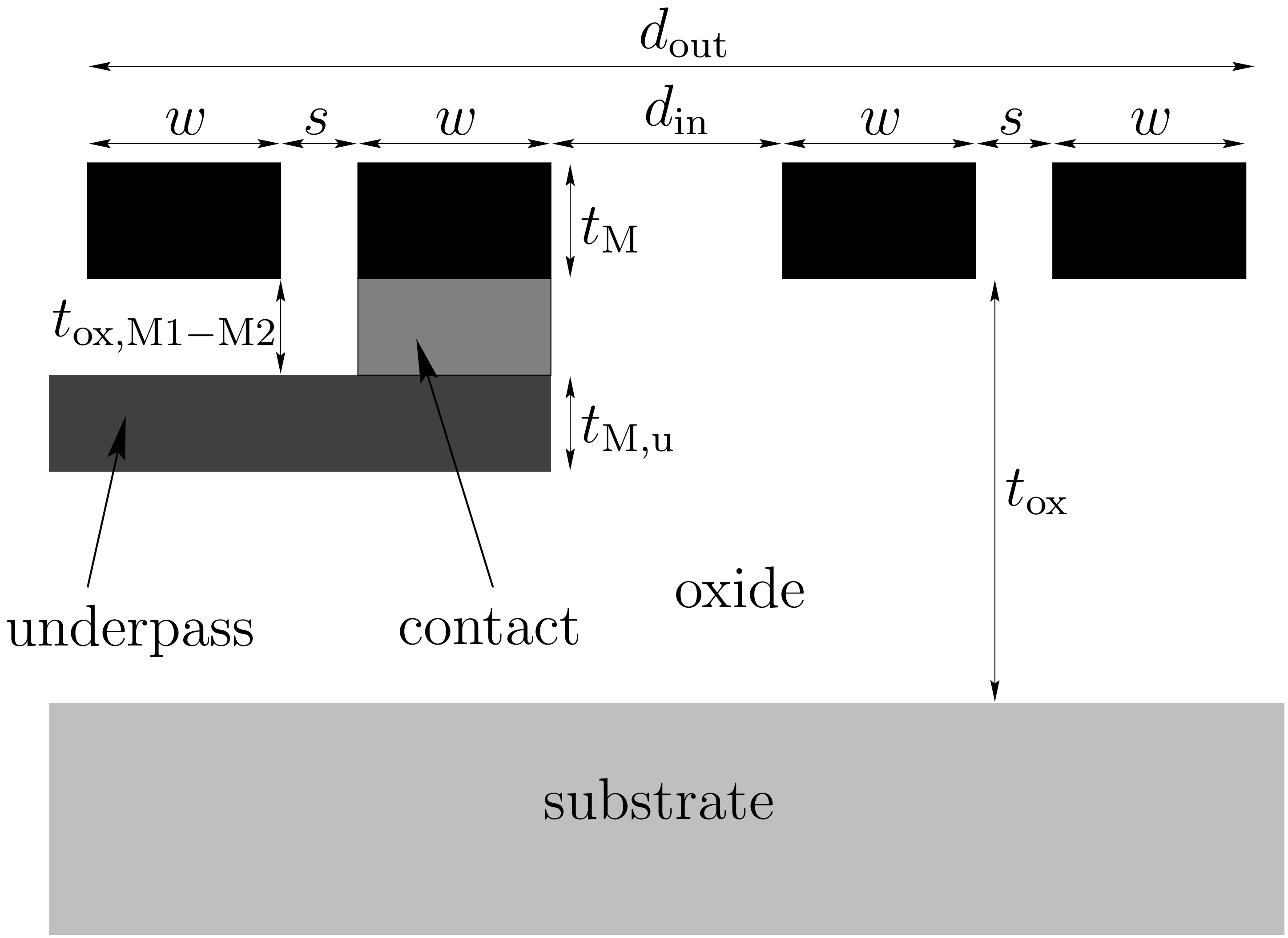
Lateral Parameters



LATERAL PARAMETERS

1. Shape: square, hexagonal, octagonal, . . .
2. Number of turns, n
3. Conductor width, w
4. Conductor spacing, s
5. d_{out}, d_{in}, $d_{\mathrm{avg}} = 0.5(d_{\mathrm{out}} + d_{\mathrm{in}})$, or $\rho = \frac{d_{\mathrm{out}} - d_{\mathrm{in}}}{d_{\mathrm{out}} + d_{\mathrm{in}}}$

Vertical Parameters



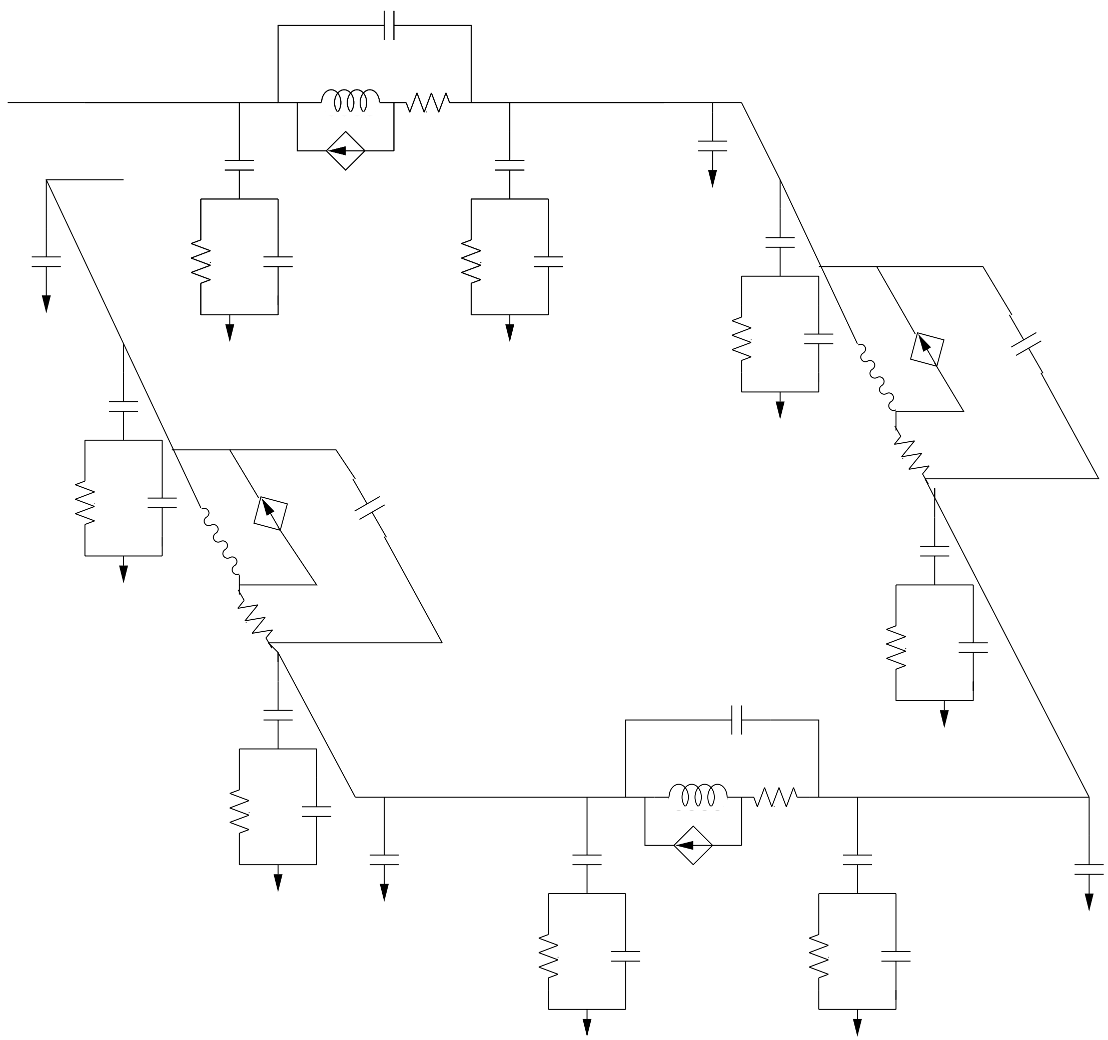
MODELING APPROACHES

- 3-D field solvers
- Segmented models
- Lumped, Scalable models

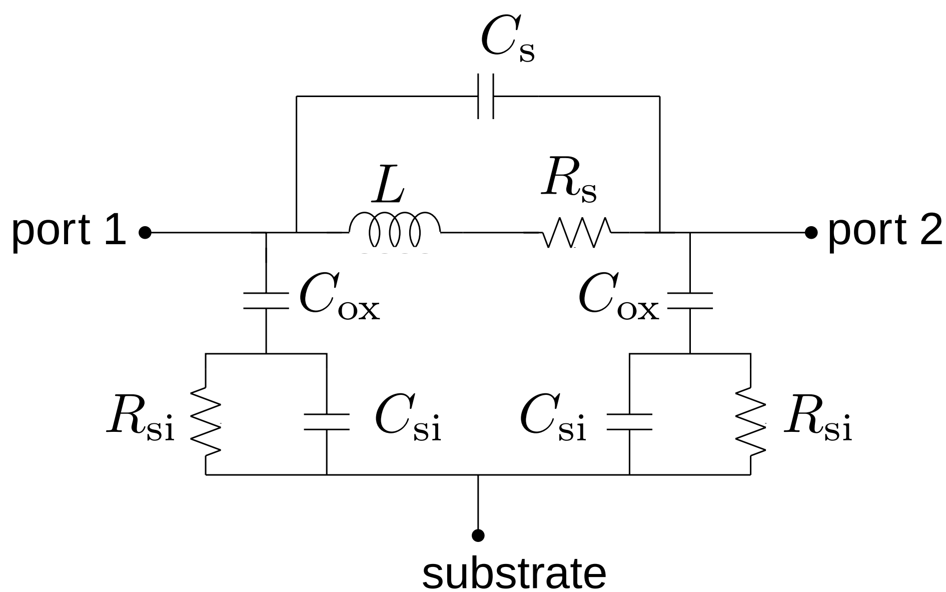
3-D FIELD SOLVERS

- General Purpose Tools
  - Solve Maxwell's equations numerically
  - **Accurate**, but **slow** and **memory intensive**
  - Examples: *Maxwell*, *MagNet*
- Custom Tools for Spiral Inductors and Transformers
  - Electrostatic and Magnetostatic approximations
  - Good for **verification**, but **inconvenient** for circuit **design** and **synthesis**
  - Examples: *ASITIC*, *SPIRAL*

SEGMENTED MODELS



LUMPED, SCALABLE MODELS



- Simple expressions for R_s, C_{ox} and C_s
- **NEED** simple, accurate expression for inductance!
- Limitations:
  - Magnetic coupling to substrate **NOT** modeled
  - Lumped approximation not valid beyond self-resonant frequency

GREENHOUSE APPROACH

Find self inductance of, and mutual inductance between every segment of spiral:

$$\mathcal{M}_{gen,i,j} = \begin{bmatrix} L_1 & M_{1,2} & \ldots & M_{1,(n-1)} & M_{1,n} \\ M_{1,2} & L_2 & \ldots & M_{2,(n-1)} & M_{2,n} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ M_{1,(n-1)} & M_{2,(n-1)} & \ldots & L_{(n-1)} & M_{n,(n-1)} \\ M_{1,n} & M_{2,n} & \ldots & M_{n,(n-1)} & L_n \end{bmatrix}$$

$$L_{\mathrm{T}} = \sum_{i=1}^{n} L_i + \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} M_{i,j}$$

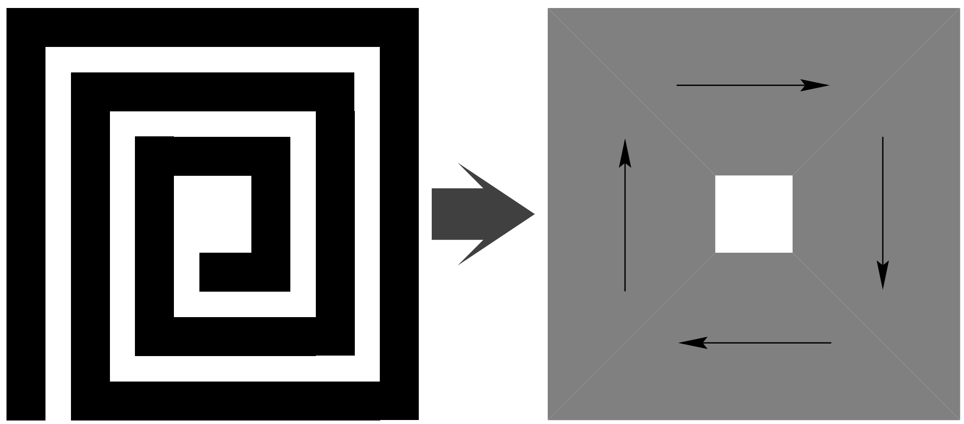
Previously Reported Expressions

$$\begin{aligned}
\text{Voorman}: \quad \mathrm{L}_{\mathrm{voo}} &= 10^{-3} n^2 d_{\mathrm{avg}} \\
\text{Dill}: \quad \mathrm{L}_{\mathrm{dil}} &= 8.5 \cdot 10^{-4} n^{5/3} d_{\mathrm{avg}} \\
\text{Bryan}: \quad \mathrm{L}_{\mathrm{bry}} &= 2.41 \cdot 10^{-3} n^{5/3} d_{\mathrm{avg}} \log(4/\rho) \\
\text{Ronkanien}: \quad \mathrm{L}_{\mathrm{ron}} &= 1.5 \mu_0 n^2 e^{-3.7(n-1)(w+s)/d_{\mathrm{out}}} \\
\text{Crols}: \quad \mathrm{L}_{\mathrm{cro}} &= 1.3 \cdot 10^{-4} (d_{\mathrm{out}}^3 / w^2) \eta_a^{5/3} \eta_w^{1/4}
\end{aligned}$$

- Empirical expressions
- Significant mean offset errors
- Even when corrected, errors $> 15 - 20\%$

DERIVATION OF ACCURATE EXPRESSIONS

- Use equivalent current sheet to simplify problem:



- Use GMD, AMD and AMSD to derive simple expression

Geometric Mean Distance (GMD)

- For distances d_1 and d_2:

$$\mathrm{GMD} = \sqrt{d_1 d_2}$$

$$\ln(\mathrm{GMD}) = \frac{1}{2}\left[\ln(d_1) + \ln(d_2)\right]$$

- For n distances:

$$\ln(\mathrm{GMD}) = \frac{1}{n}\left[\ln(d_1) + \ln(d_2) \cdots + \ln(d_{\mathrm{n}})\right]$$

GMD in Inductance Calculations

- Need to evaluate of GMD of conductor cross-section(s):
  - Self: GMD of conductor cross-section from itself
  - Mutual: GMD between two conductor cross-sections
- Use continuous variable definition of GMD
  - Need integrals rather than sums
- GMD introduced in to inductance calculations by J. C. Maxwell

GMD in Inductance Calculations

- For cross sections in one dimension (current sheets):

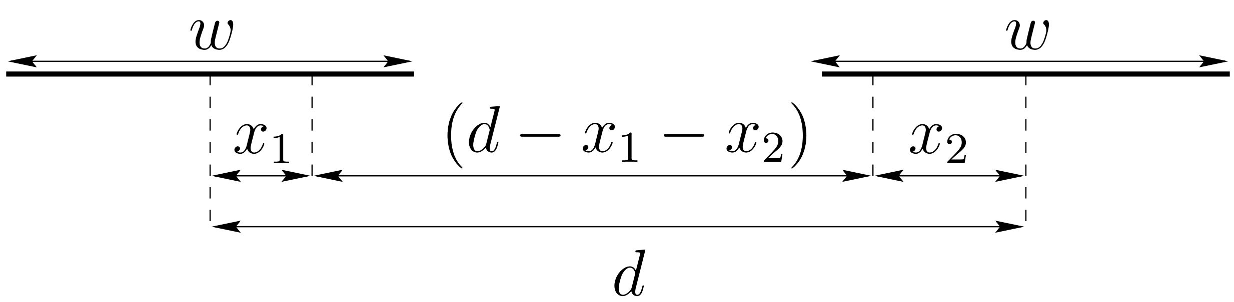
$$l_1 l_2 \ln(\mathrm{GMD}) = \iint \ln(r)\, dx\, dx'$$

  – l_1 and l_2 are the lengths of the cross-sections

  – dx and dx' are the elements of the cross-sections

  – r is the distance between the elements

GMD between Two Lines

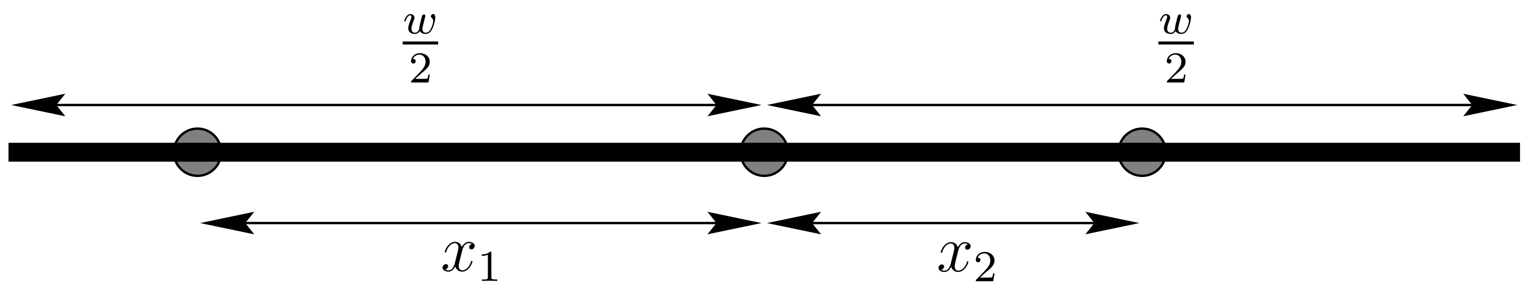


$$\ln(\mathrm{GMD}) = \frac{1}{w^2}\int_{-0.5w}^{0.5w}\int_{-0.5w}^{0.5w} \ln|d - x_1 - x_2|\,dx_1 dx_2$$

$$\approx \ln(d) - \frac{w^2}{12d^2} - \frac{w^4}{60d^4}\ldots$$

- Basis for mutual inductance calculations in Greenhouse method

GMD, AMD AND AMSD OF A LINE

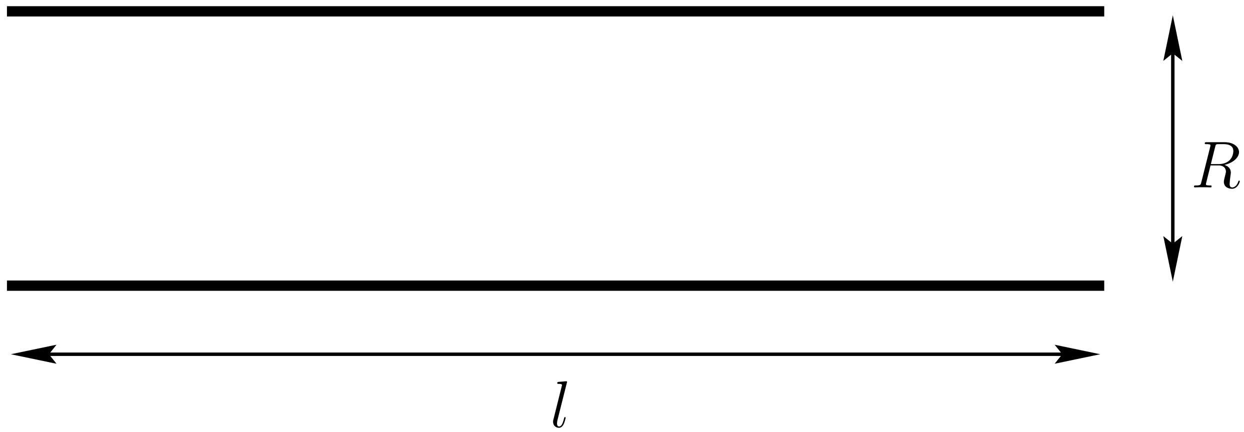


$$\ln(\text{GMD}) = \frac{1}{w^2}\int_{-0.5w}^{0.5w}\int_{-0.5w}^{0.5w} \ln|x_1 + x_2| dx_1 dx_2 = \ln(w) - 1.5$$

$$\text{AMD} = \frac{1}{w^2}\int_{-0.5w}^{0.5w}\int_{-0.5w}^{0.5w} |x_1 + x_2| dx_1 dx_2 = \frac{w}{3}$$

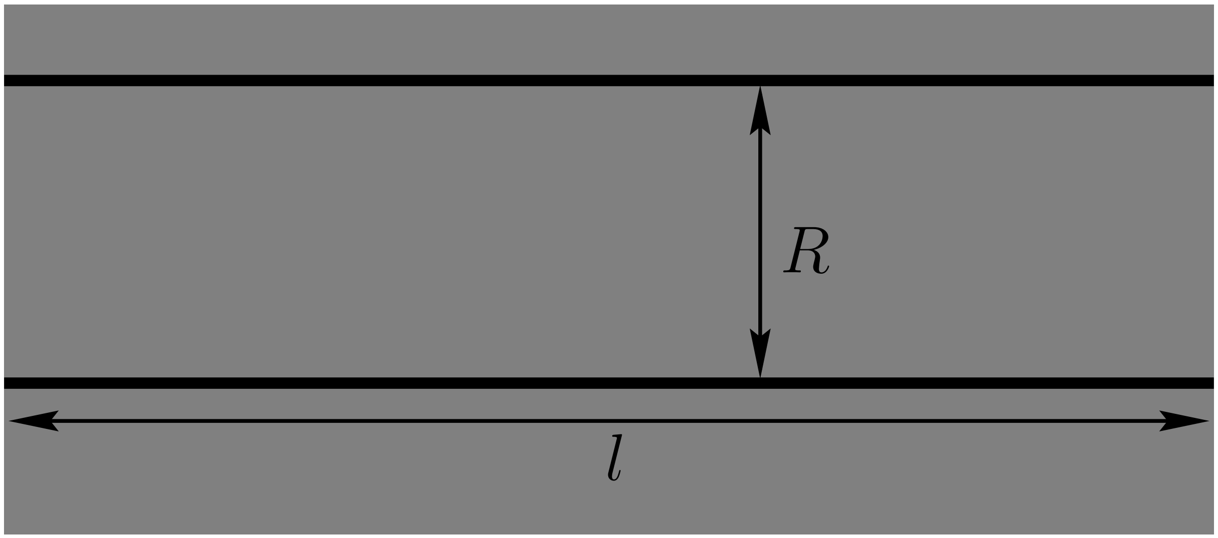
$$\text{AMSD}^2 = \frac{1}{w^2}\int_{-0.5w}^{0.5w}\int_{-0.5w}^{0.5w} |x_1 + x_2|^2 dx_1 dx_2 = \frac{w^2}{6}$$

Parallel Lines of Equal Length



$$M \approx \frac{\mu l}{2\pi}\left[\ln(2l) - \ln(R) - 1 + \frac{R}{l} - \frac{R^2}{4l^2}\right] \text{ for } \left(\frac{R}{l}\right) \leq 1$$

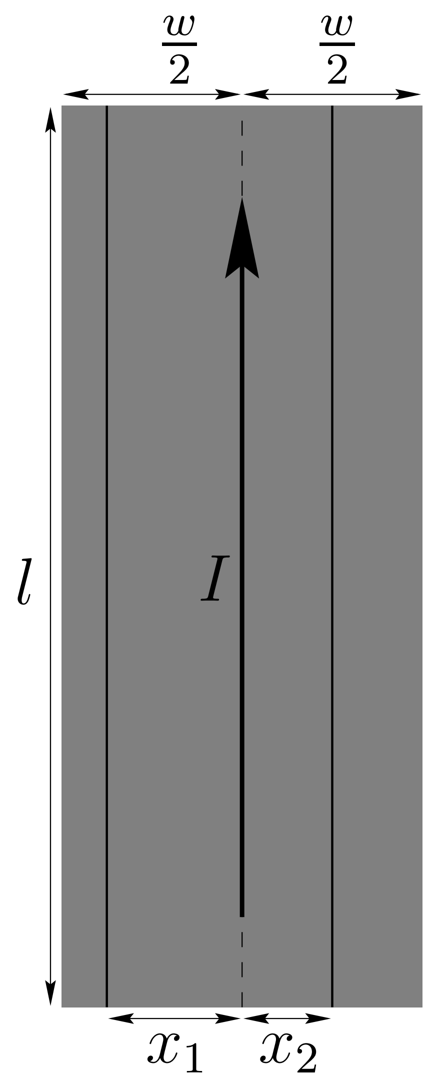
Inductance of Current Sheet



$$M = \frac{\mu l}{2\pi}\left[\ln(2l) - \ln(R) - 1 + \frac{R}{l} - \frac{R^2}{4l^2}\right]$$

$$L_{\mathrm{s}} = \frac{\mu l}{2\pi}\left[\ln(2l) - \ln(\mathrm{GMD}) - 1 + \frac{\mathrm{AMD}}{l} - \frac{\mathrm{AMSD}^2}{4l^2}\right]$$

INDUCTANCE OF RECTANGULAR CURRENT SHEET

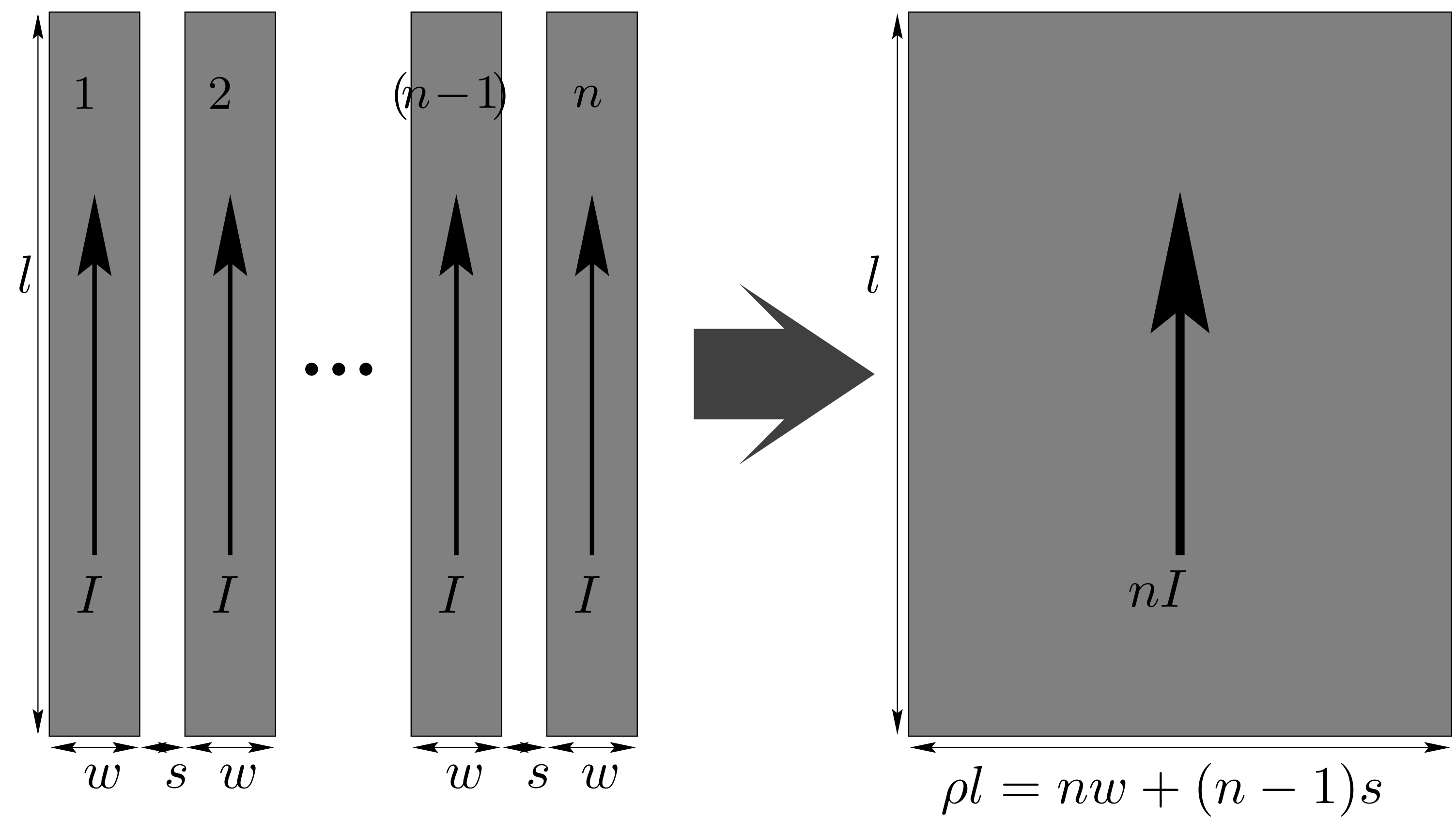


$$-\frac{w}{2} < x_1, x_2 < \frac{w}{2}$$

$$\begin{aligned} \ln(\text{GMD}) &= \overline{\ln|x_1 + x_2|} &= \ln w - 1.5 \\ \text{AMD} &= \overline{|x_1 + x_2|} &= \frac{w}{3} \\ \text{AMSD}^2 &= \overline{(x_1 + x_2)^2} &= \frac{w^2}{6} \end{aligned}$$

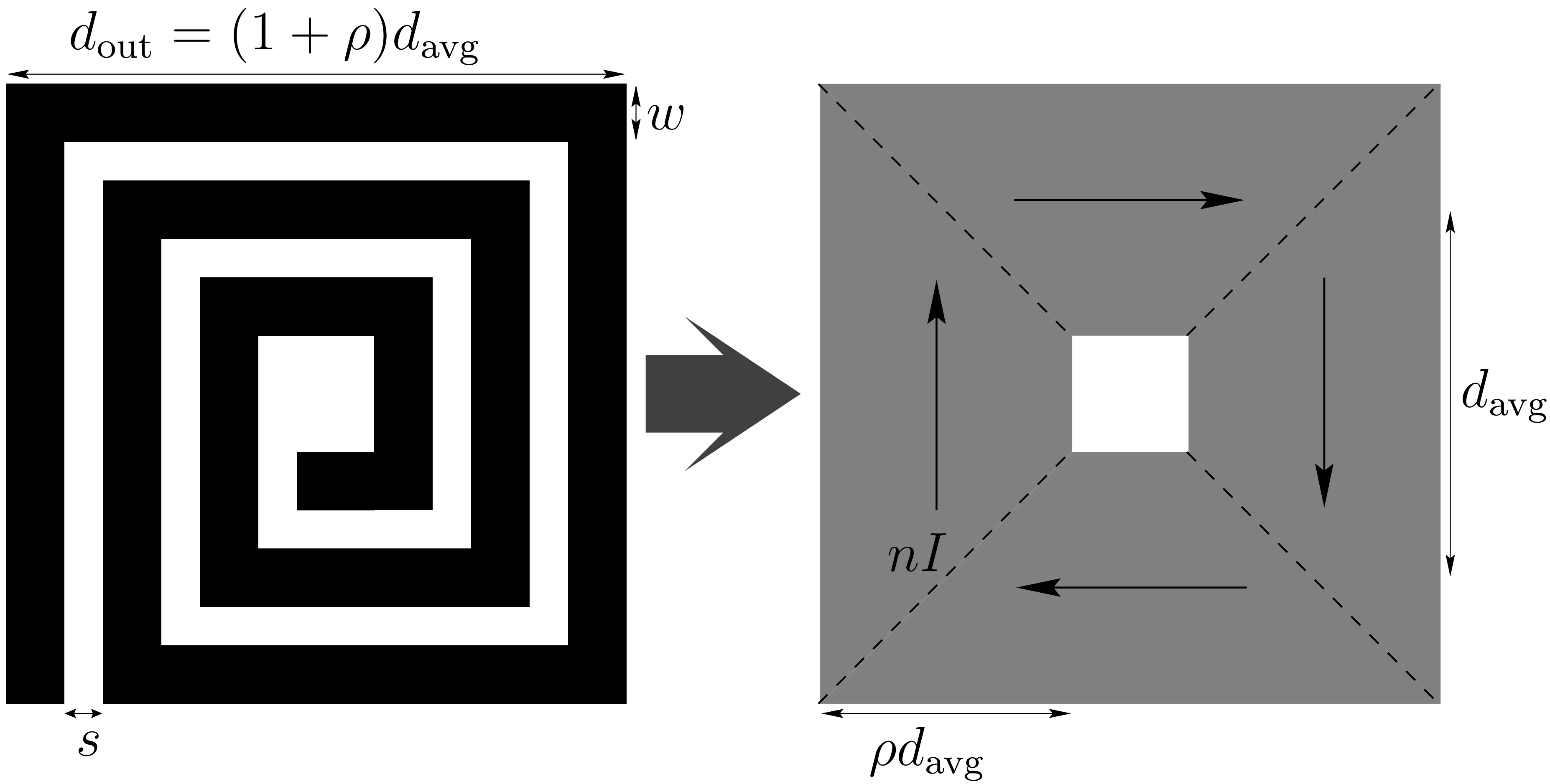
$$L = \frac{\mu l}{2\pi}\left[\ln\left(\frac{2l}{w}\right) + 0.5 + \frac{w}{3l} - \frac{w^2}{24l^2}\right]$$

Equivalent Rectangular Current Sheet

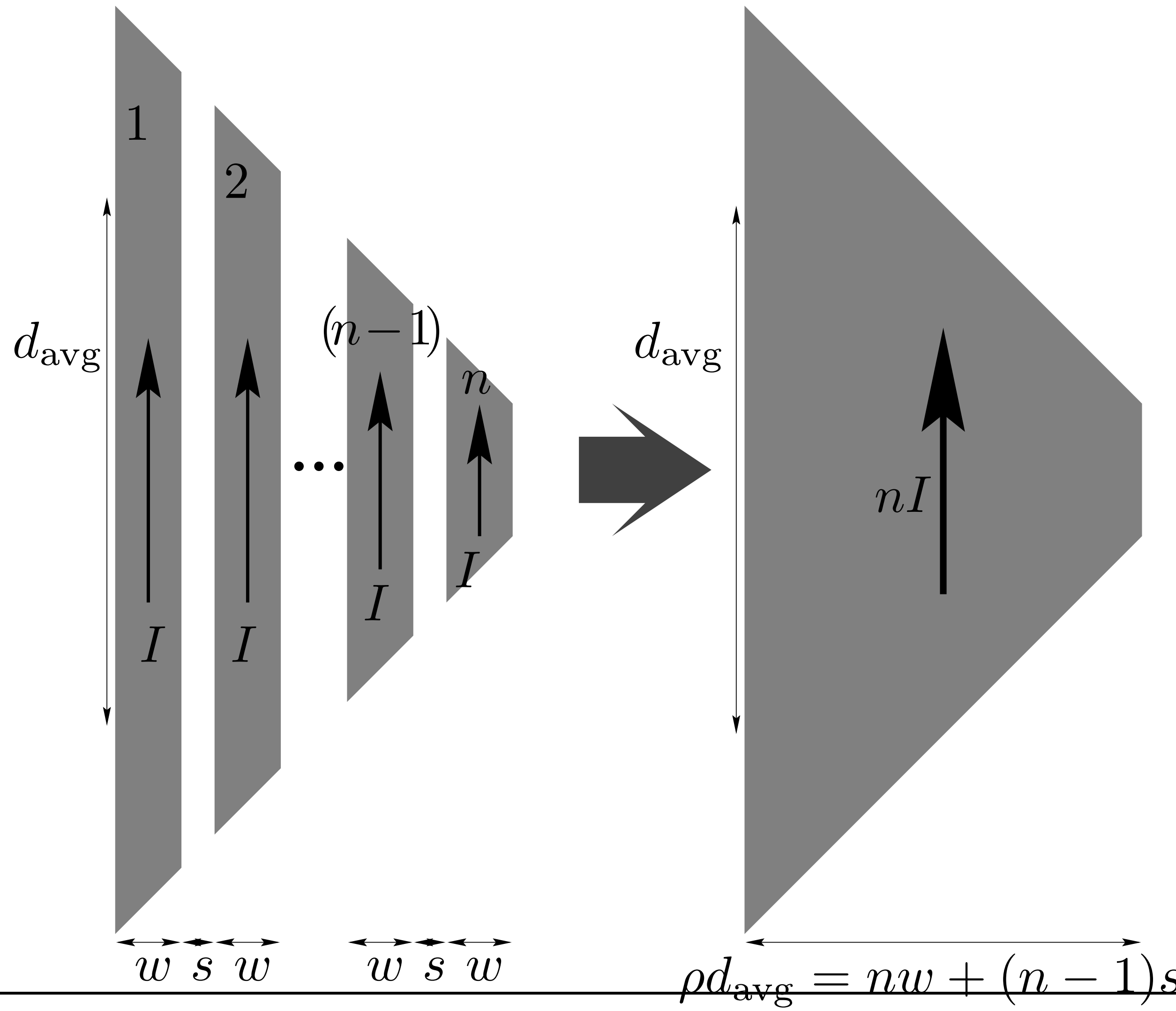


$$L = \frac{\mu n^2 l}{2\pi}\left[\ln\left(\frac{2}{\rho}\right) + 0.5 + \frac{\rho}{3} - \frac{\rho^2}{24}\right]$$

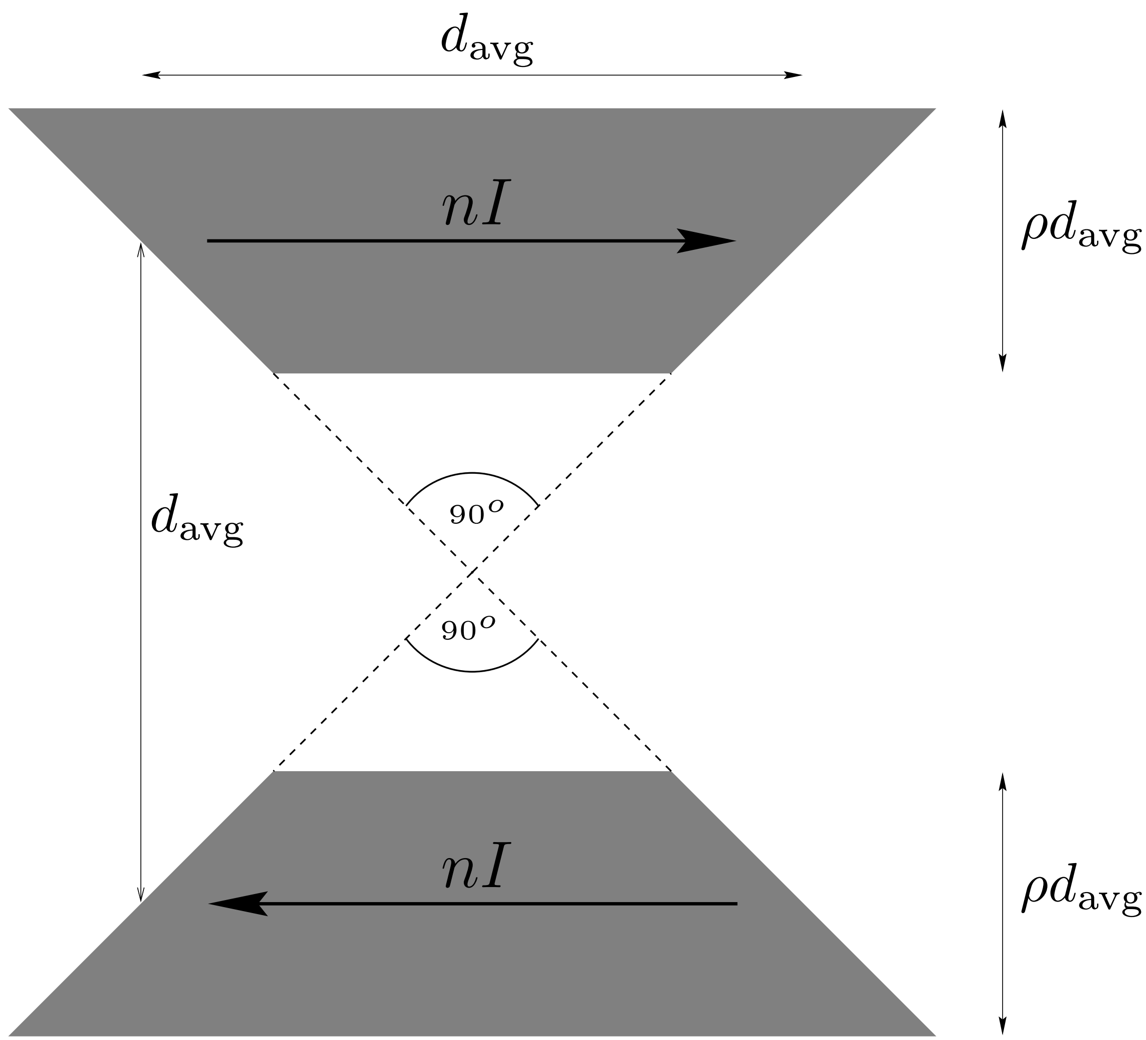
Approximating a Square Spiral



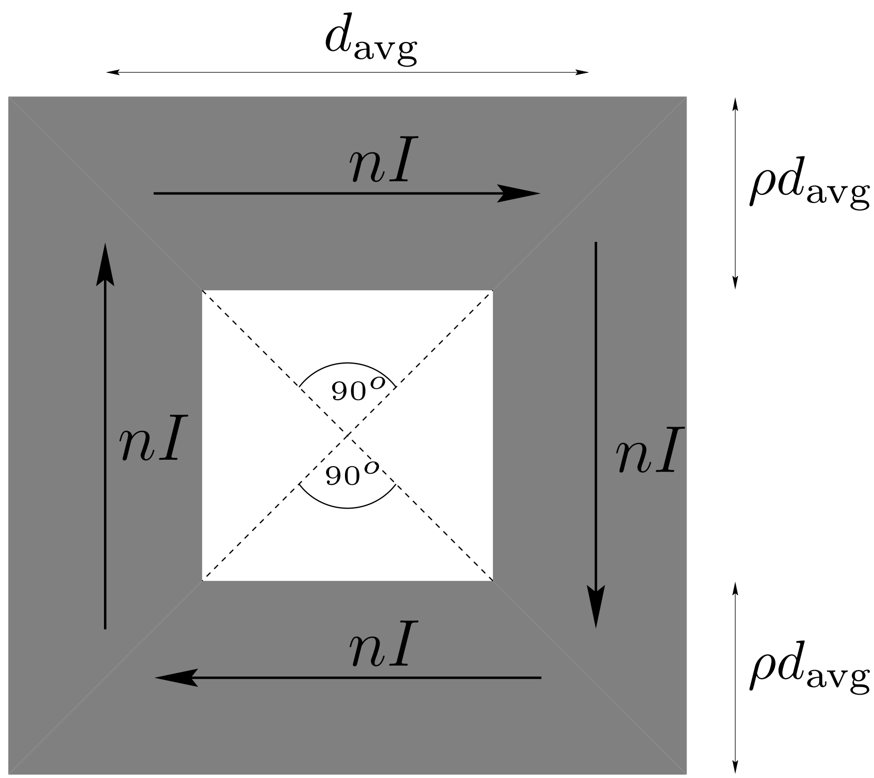
One Side of a Square Spiral: L_S



OPPOSITE SIDES OF A SQUARE SPIRAL: M_{opp}

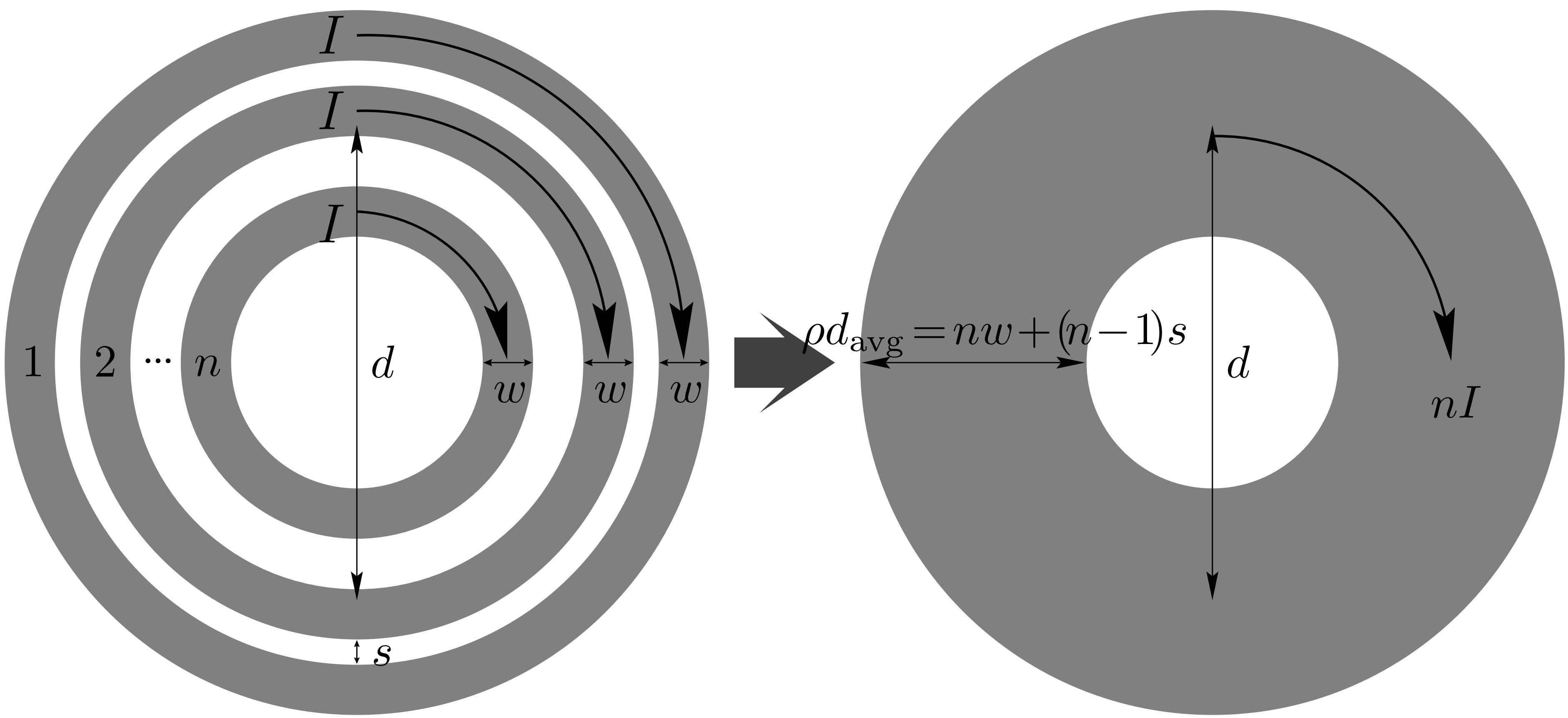


Current Sheet Expression for a Square Spiral



$$\begin{aligned} L_{\text{sq}} &= 4(L_{\text{s}} + M_{\text{opp}}) \\ &= \frac{2\mu n^2 d_{\text{avg}}}{\pi}\left[\ln\left(\frac{2.067}{\rho}\right) + 0.178\rho + 0.125\rho^2\right] \end{aligned}$$

Concentric Circular Conductors



$$L \approx \frac{\mu n^2 d_{\text{avg}}}{2}\left[\ln\left(\frac{1}{\rho}\right) + 0.9 + 0.2\rho^2\right]$$

Current Sheet Expressions

$$L_{\mathrm{cursh}} = \frac{\mu n^2 d_{\mathrm{avg}} c_1}{2} \left[\ln(c_2/\rho) + c_3\rho + c_4\rho^2 \right]$$

Layout	c_1	c_2	c_3	c_4
Square	1.27	2.07	0.18	0.13
Hexagonal	1.09	2.23	0.00	0.17
Octagonal	1.07	2.29	0.00	0.19
Circle	1.00	2.46	0.00	0.20

Other Inductance Expressions

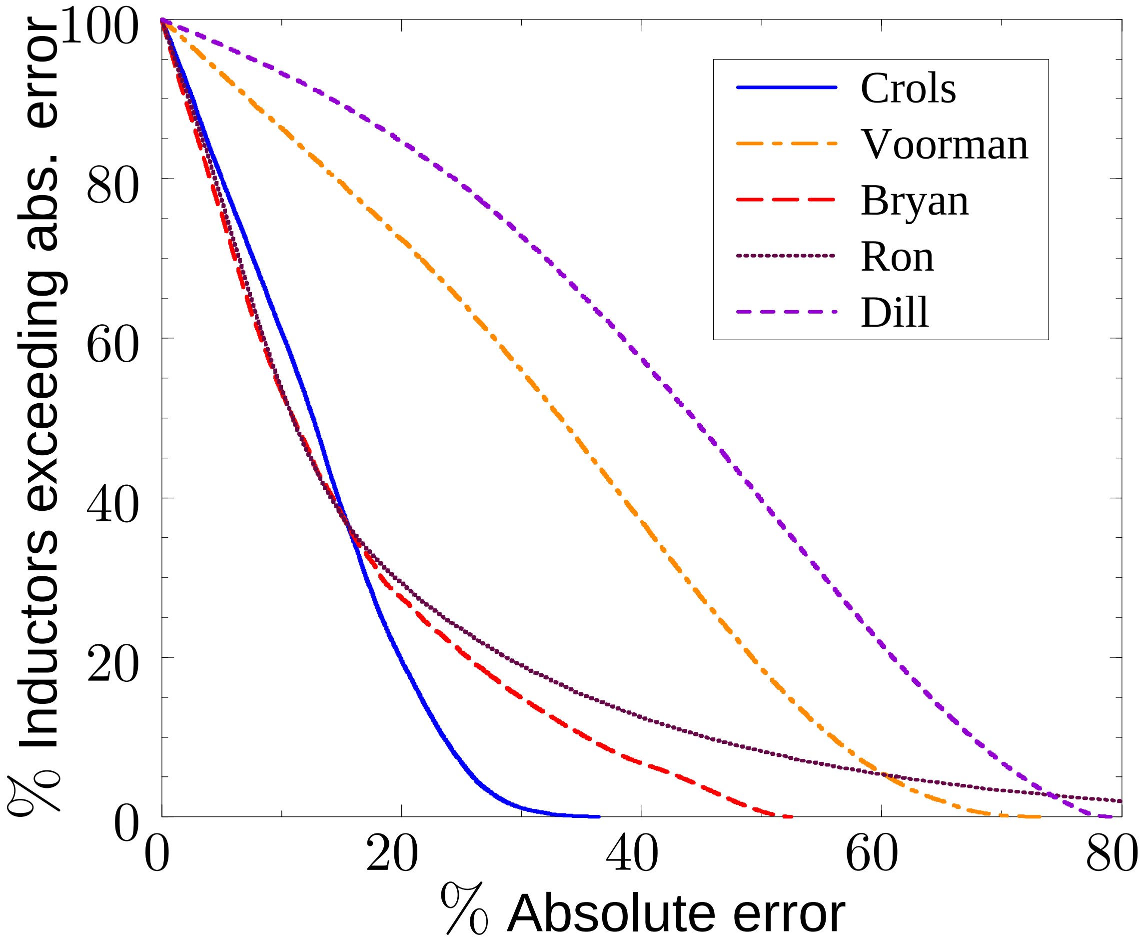
- Monomial Expression :

$$L_{\text{mon}} = \beta d_{\text{out}}^{\alpha_1} w^{\alpha_2} d_{\text{avg}}^{\alpha_3} n^{\alpha_4} s^{\alpha_5}$$

- Modified Wheeler Expression :

$$L_{\text{mw}} = K_1 \mu_0 \frac{n^2 d_{\text{avg}}}{1 + K_2 \rho}$$

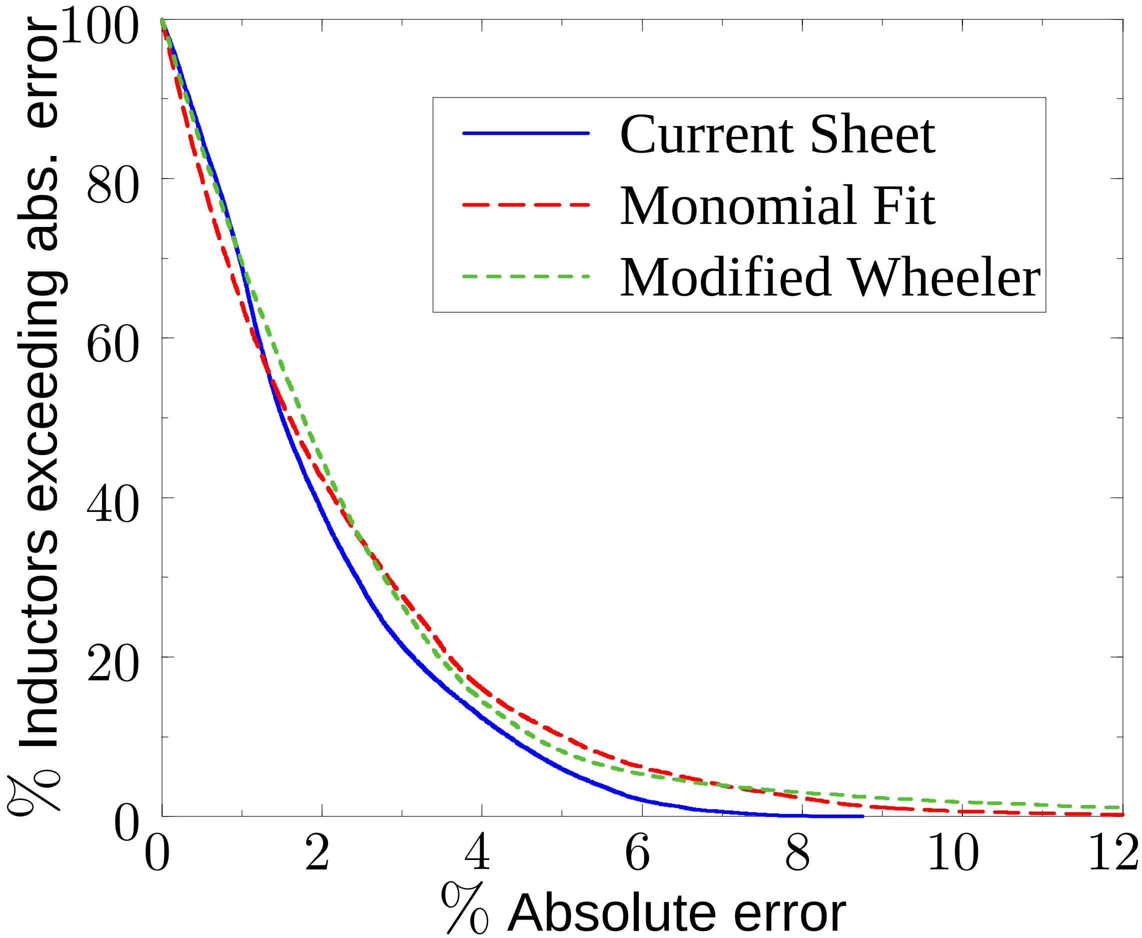
Comparison to Field Solvers: Previous Work



	Min	Max
$L(\text{nH})$	0.1	70
$\text{OD}(\mu\text{m})$	100	400
n	1	20
s/w	0.02	3
ρ	0.03	0.95

$19,000$ simulations

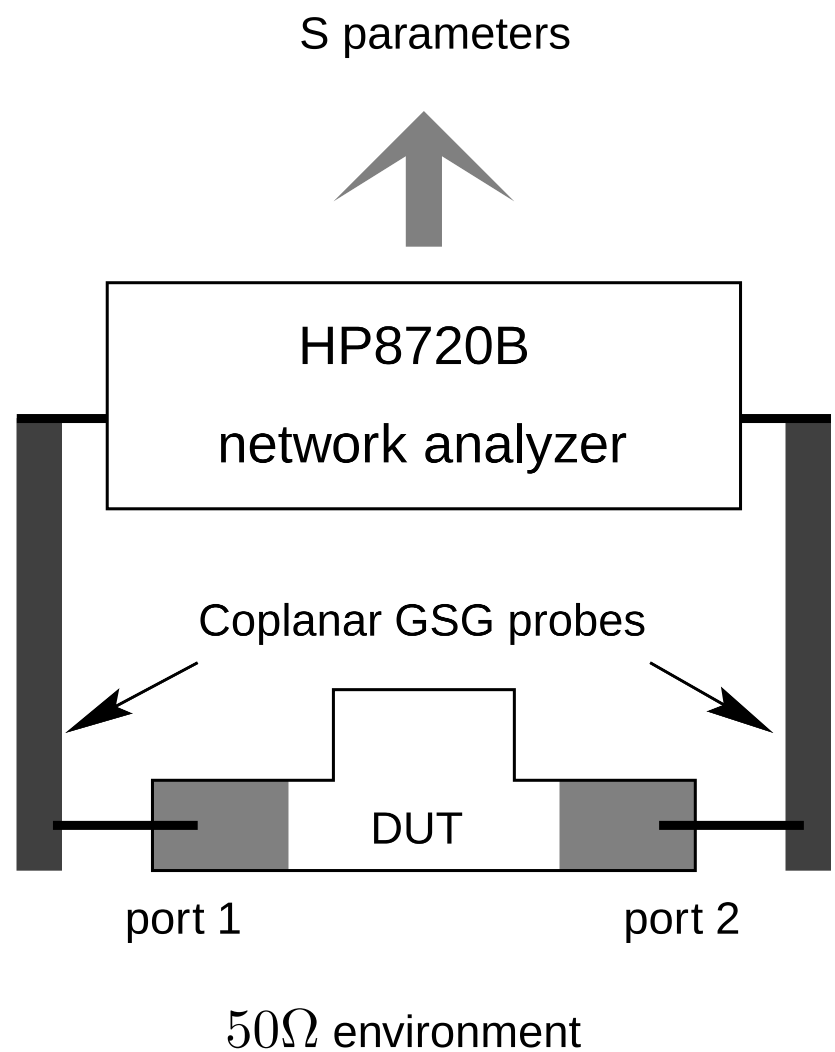
COMPARISON TO FIELD SOLVERS: NEW WORK



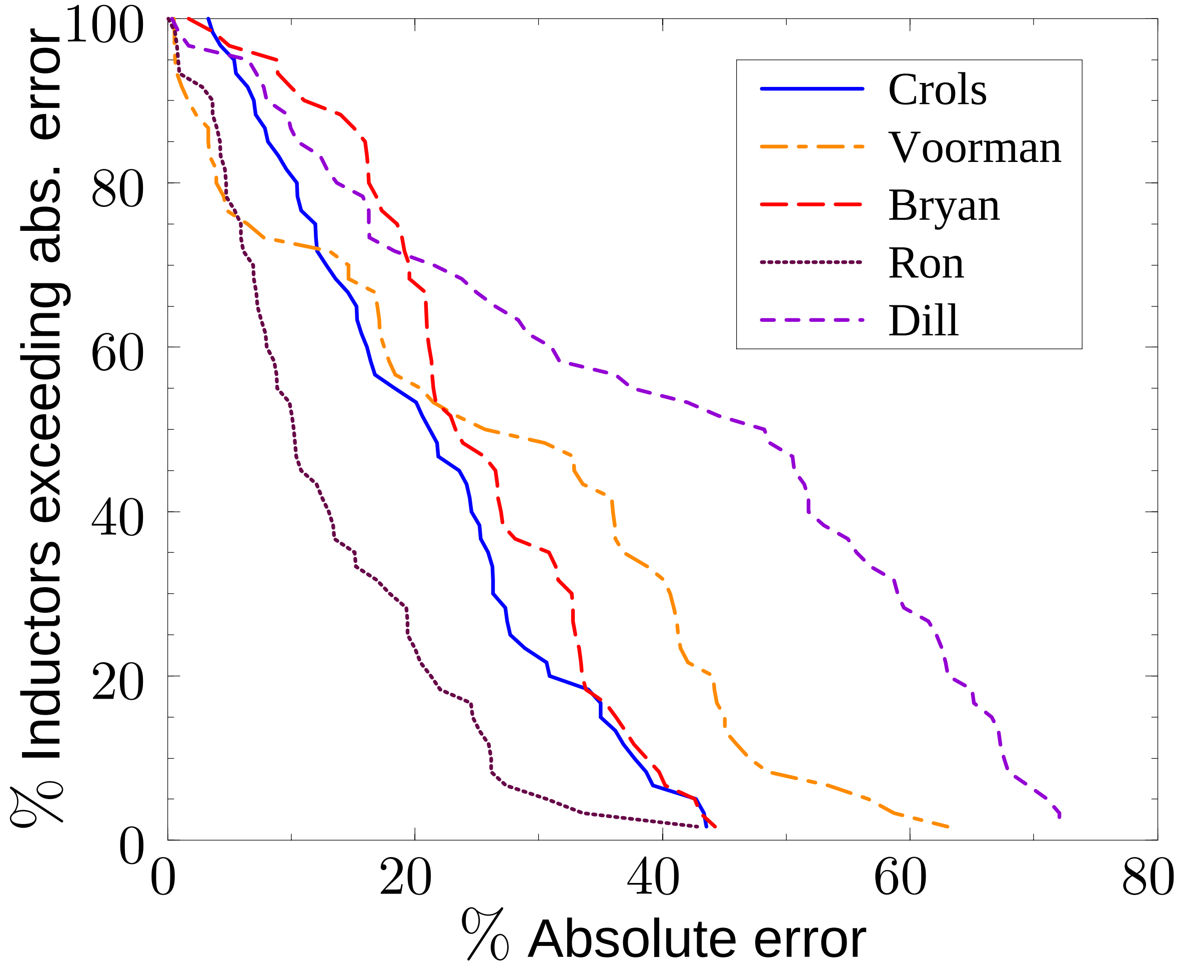
	Min	Max
L(nH)	0.1	70
OD(μm)	100	400
n	1	20
s/w	0.02	3
ρ	0.03	0.95

$19,000$ simulations

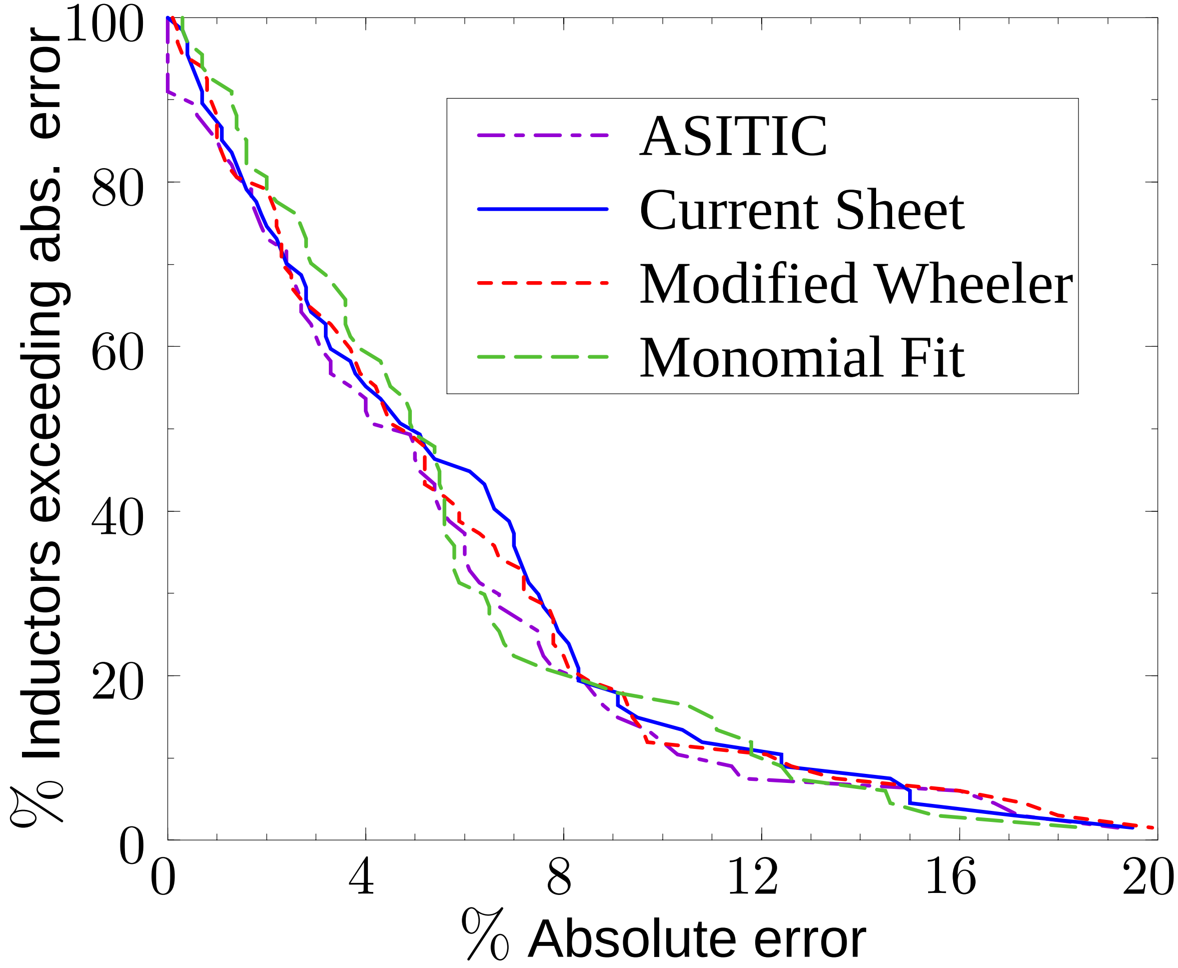
Experimental Set-up



Comparison to Experiments: Previous Work



COMPARISON TO EXPERIMENTS: NEW WORK



PARAMETERS OF INTEREST

- Inductor quality factor (Q_{L})

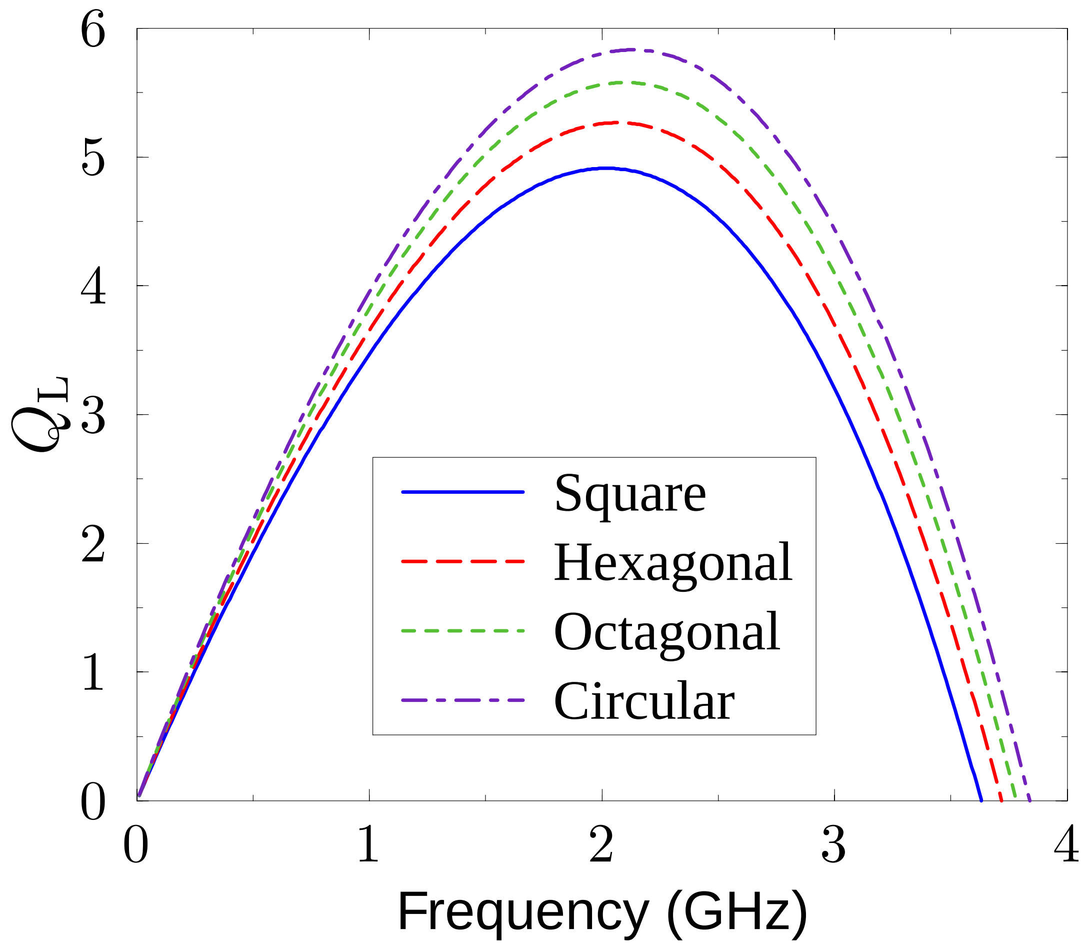
$$Q_L = 2\pi \frac{[\text{peak magnetic energy} - \text{peak electric energy}]}{\text{energy loss in one oscillation cycle}}$$

- Tank quality factor (Q_{tank})

$$Q_{\mathrm{tank}} = 2\pi \frac{\text{peak magnetic energy}}{\text{energy loss in one oscillation cycle}}$$

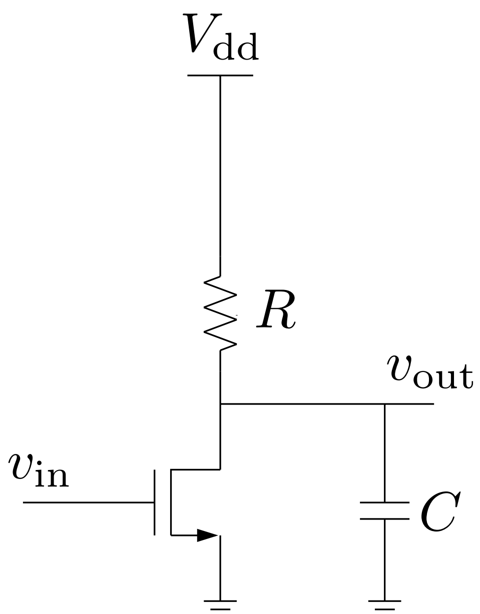
- Self-resonance frequency (ω_{res}), frequency at which $Q_{\mathrm{L}} = 0$

EXAMPLE: MAXIMUM Q_{L} @ 2GHz FOR $L = 8\mathrm{nH}$

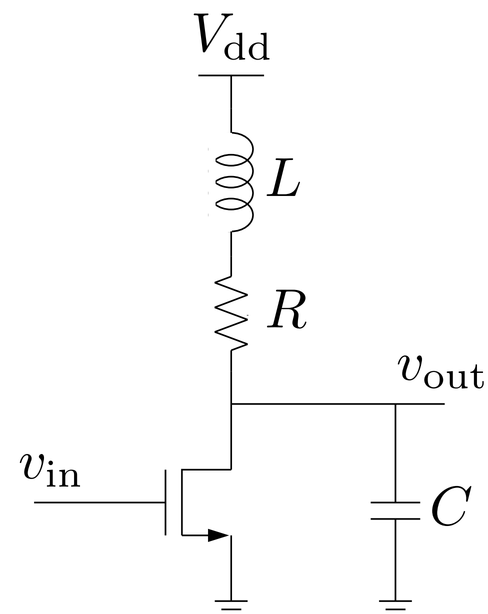


EXAMPLE: SHUNT-PEAKED AMPLIFIER

Common Source Amplifier

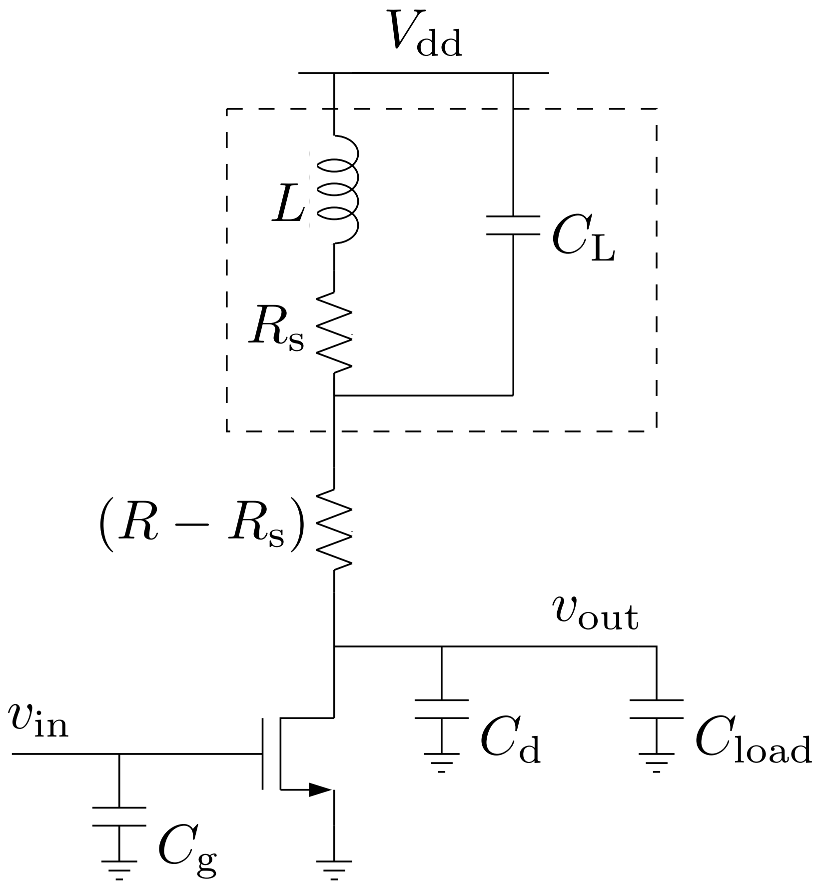


Shunt-peaked Amplifier



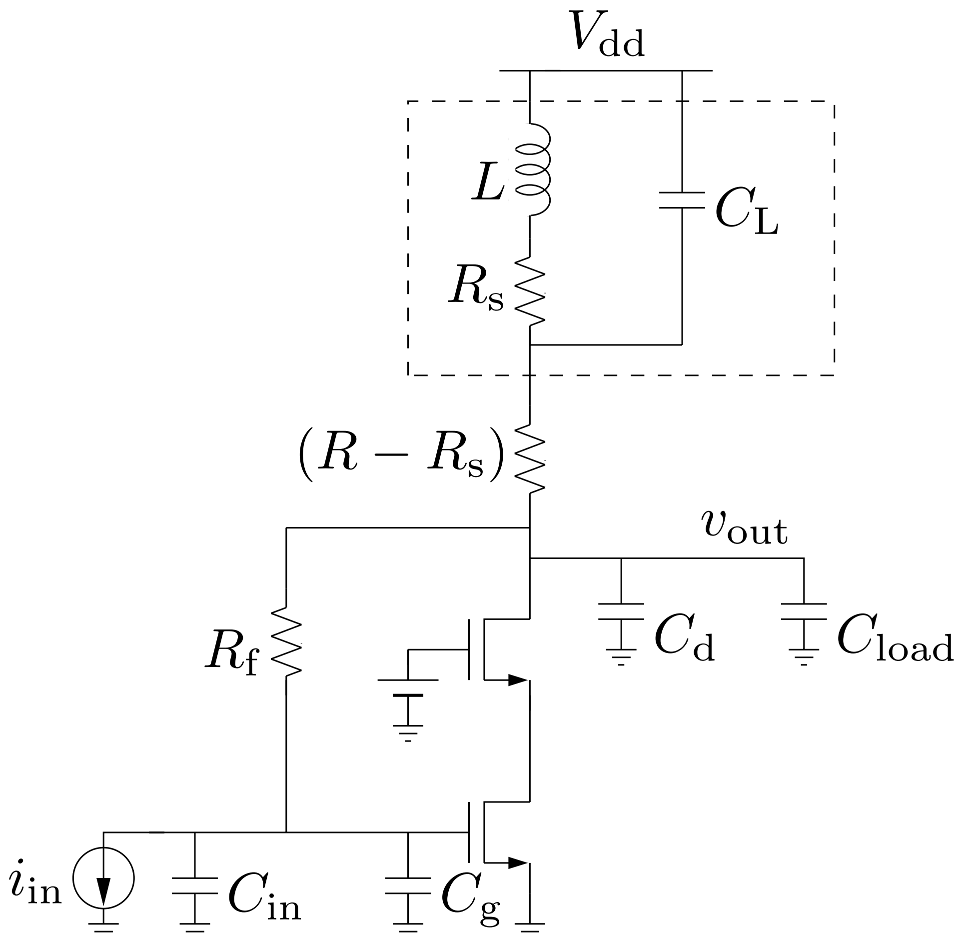
- Bandwidth enhancement using zeros
- No additional power dissipation

ON-CHIP SHUNT PEAKING



- Work with inductor parasitics
- R_s is **not** an issue (now part of load resistance)
- Inductor Q is **not** relevant
- Minimize area and C_L
- L determined by R, C_{load}, C_L and C_d

Shunt-Peaked Transimpedance Amplifier

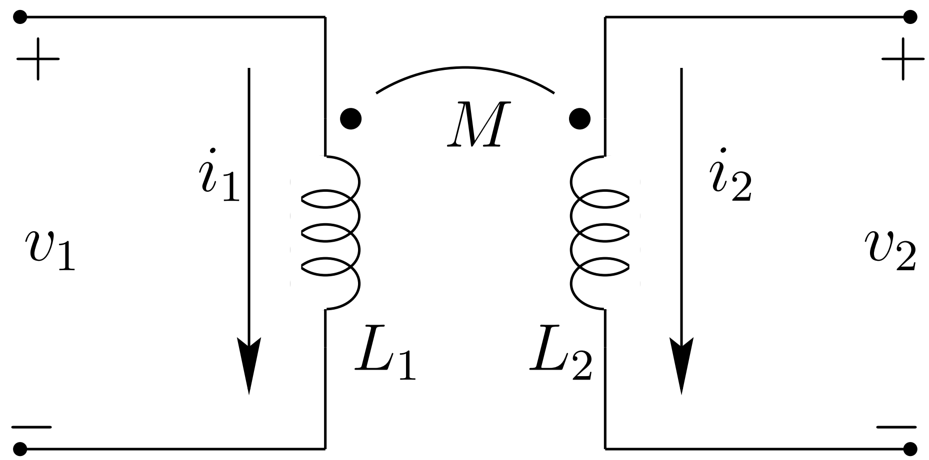


- Input current drive
- Cascode stage
- On-chip shunt-peaking
- Feedback

Design Methodology

1. Design and optimize transimpedance stage without shunt peaking
2. Transistor current determines conductor width, w
3. Lithography sets spacing, s
4. Choose n and AD to realize desired L while minimizing parasitic capacitance and area
5. Maximize transimpedance resistance, R_{f}

TRANSFORMER

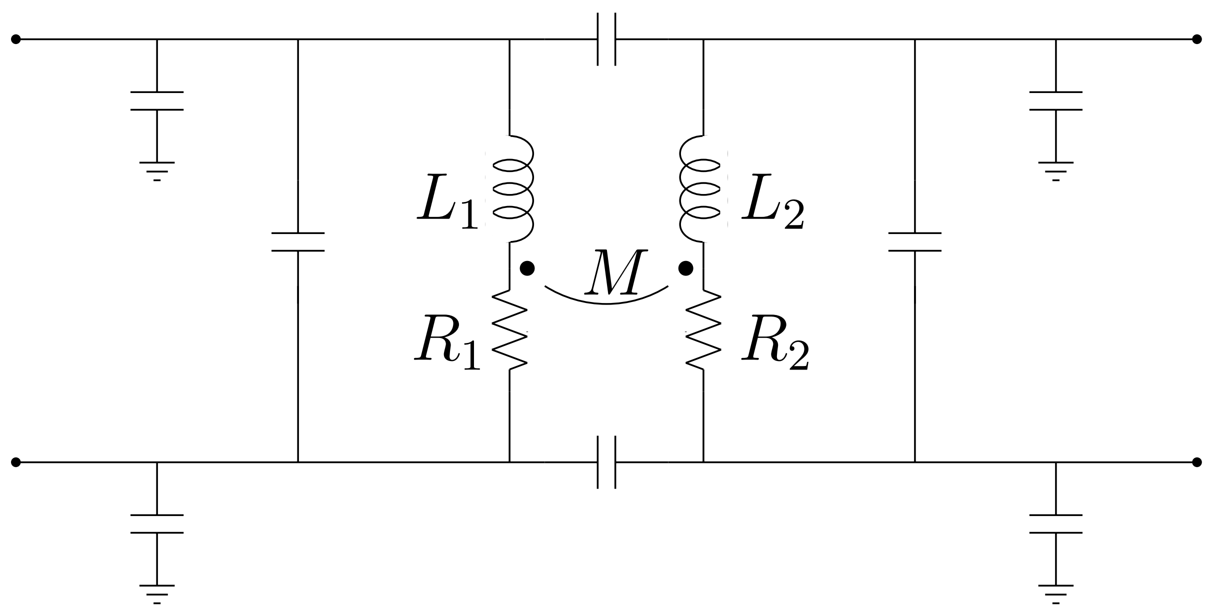


- $v_1 = L_1 \frac{\partial i_1}{\partial t} + M \frac{\partial i_2}{\partial t}$

  $v_2 = L_2 \frac{\partial i_2}{\partial t} + M \frac{\partial i_1}{\partial t}$

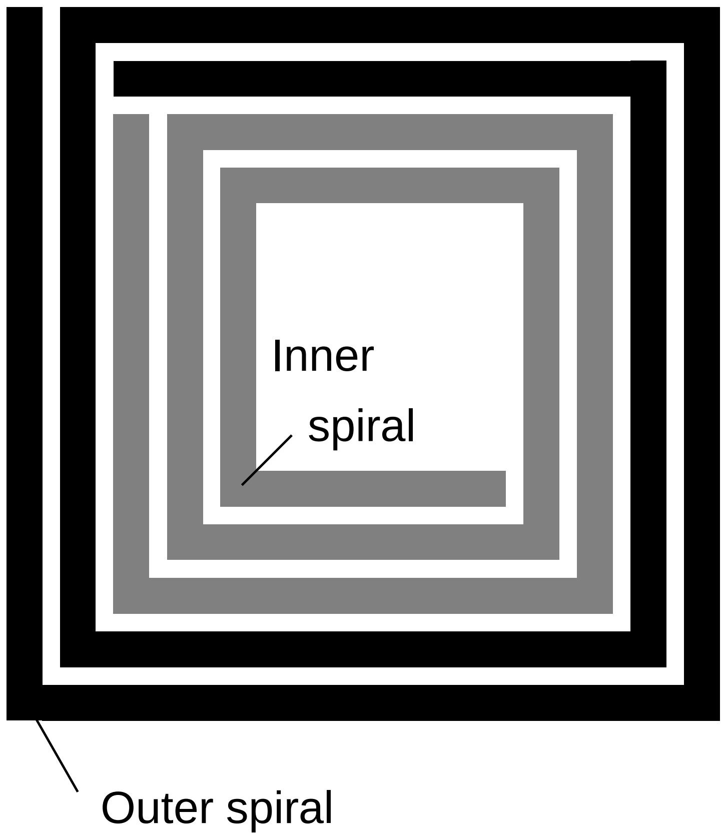
- Mutual coupling coefficient, $k = \frac{M}{\sqrt{L_1 L_2}}$
- $|k| \leq 1$

NON-IDEAL TRANSFORMER



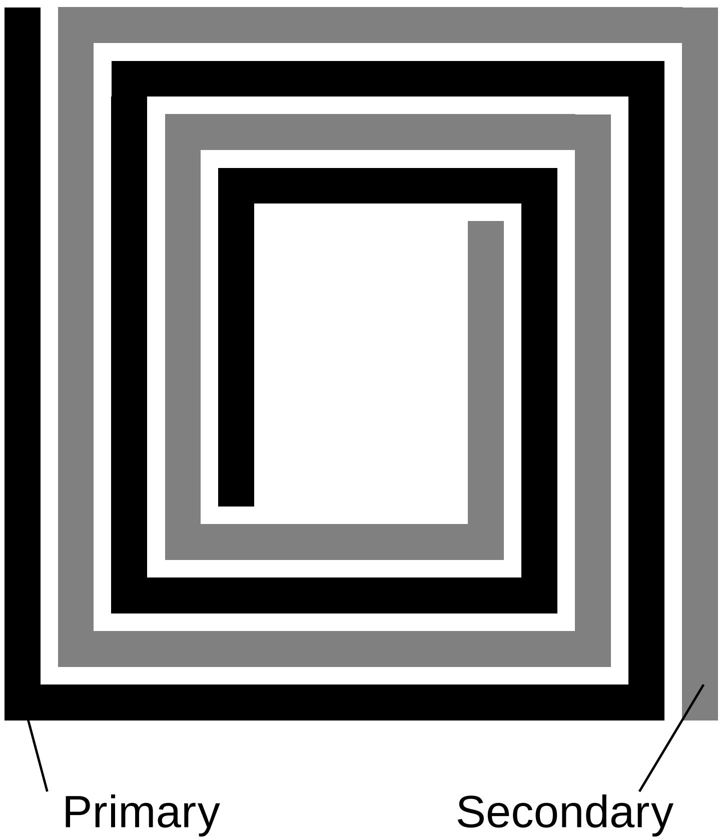
- $k = \frac{M}{\sqrt{L_1 L_2}} < 1.$
- Series resistance.
- Port-to-port & port-to-substrate capacitances

TAPPED TRANSFORMER



- **advantages:**
  - High L_1, L_2
  - Top metal layer
  - Low port-to-port capacitance
- **disadvantages:**
  - Asymmetric
  - Low $k(\approx 0.3 - 0.5)$

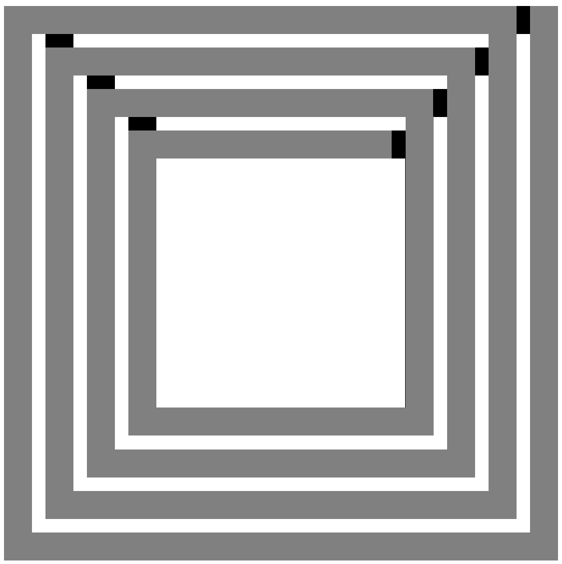
INTERLEAVED TRANSFORMER



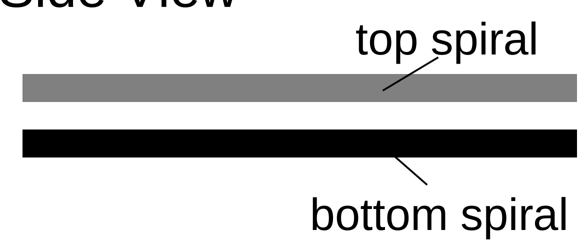
- **advantages:**
  - Medium k $(\approx 0.7 - 0.8)$
  - Symmetric
  - Top metal layer
- **disadvantages:**
  - Medium port-to-port capacitance
  - Low L_1, L_2

Stacked Transformer

Top View

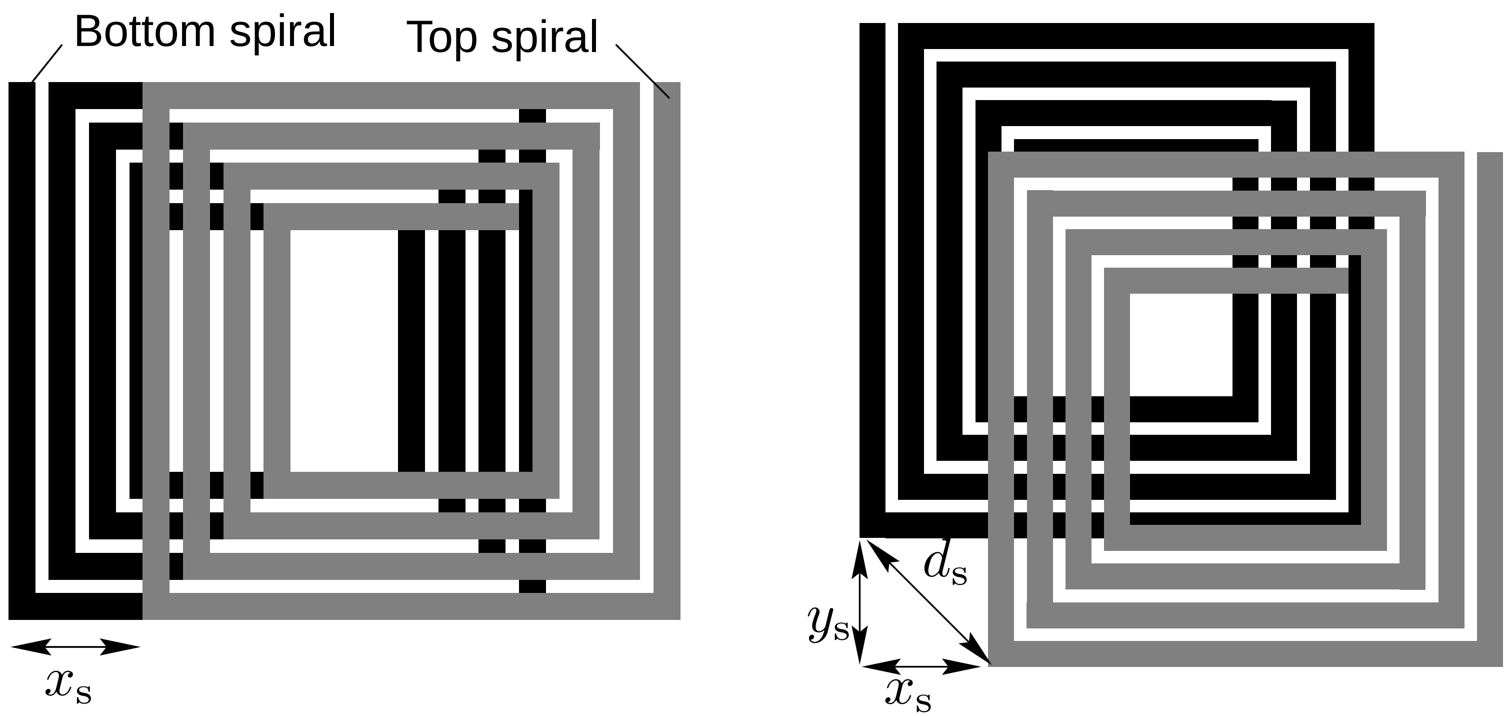


Side View



- **advantages:**
  - High $k (\approx 0.9)$
  - High L_1, L_2
  - Area efficient
- **disadvantages:**
  - Multiple metal layers
  - High port-to-port & port-to-substrate capacitances

STACKED TRANSFORMER VARIATIONS



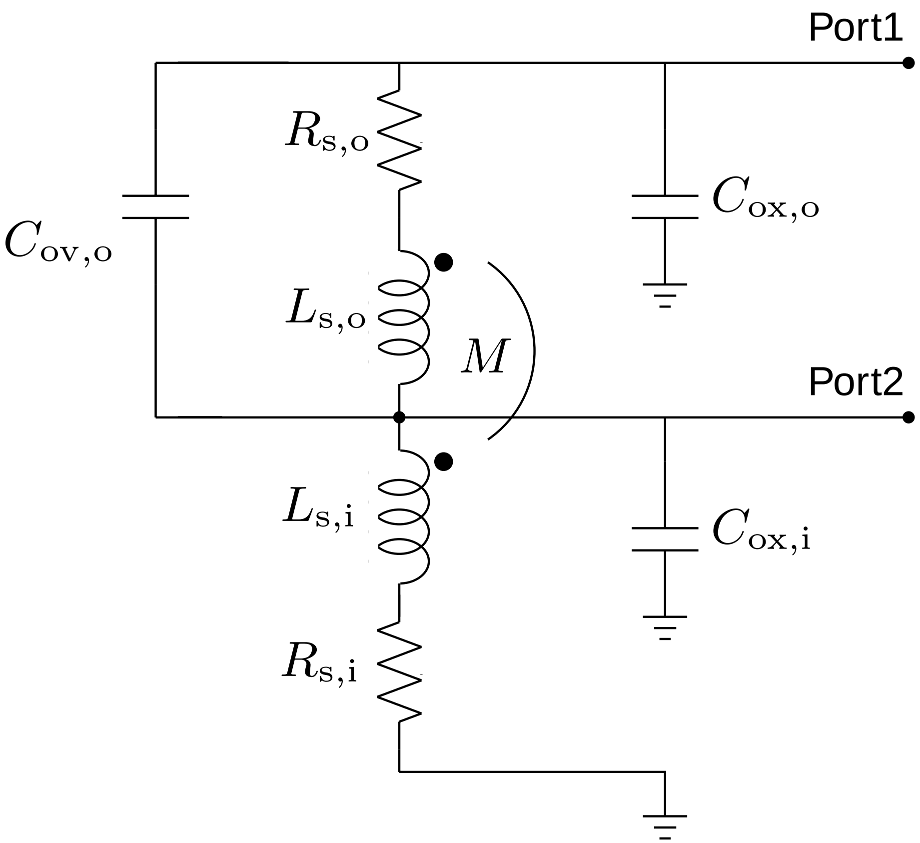
- Shift top and bottom spirals laterally or diagonally
- Trade-off lower k for reduced port-to-port capacitance

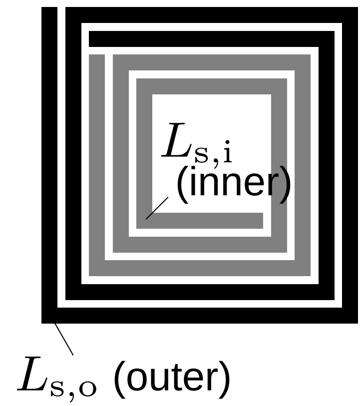
Comparison of Transformer Realizations

Transformer type	Area	Coupling coefficient, k	Self-inductance	Self-resonant frequency
Tapped	High	Low	Mid	High
Interleaved	High	Mid	Low	High
Stacked	Low	High	High	Low

- Non-idealities result in trade-offs
- Optimal choice determined by circuit application
- Transformer **models** needed for comparison

TAPPED TRANSFORMER MODEL

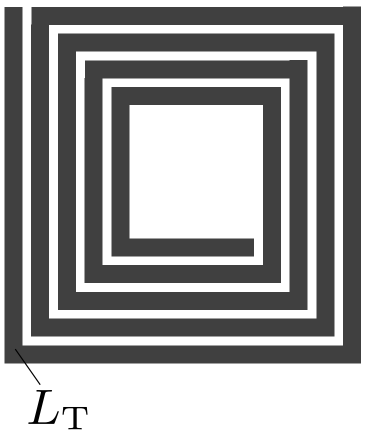




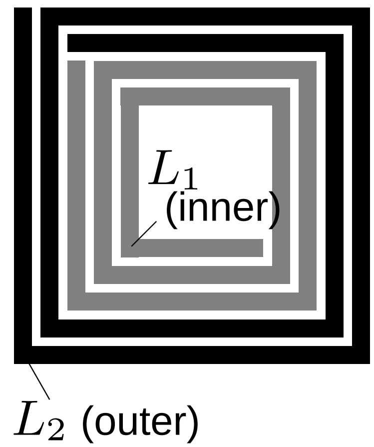
- Evaluate $C_{ov,o}$, $C_{ox,o}$, $C_{ox,i}$, $R_{s,o}$ & $R_{s,i}$ by extending previous work
- Use inductance expression for $L_{s,o}$, $L_{s,i}$
- Calculate M

MUTUAL INDUCTANCE CALCULATION

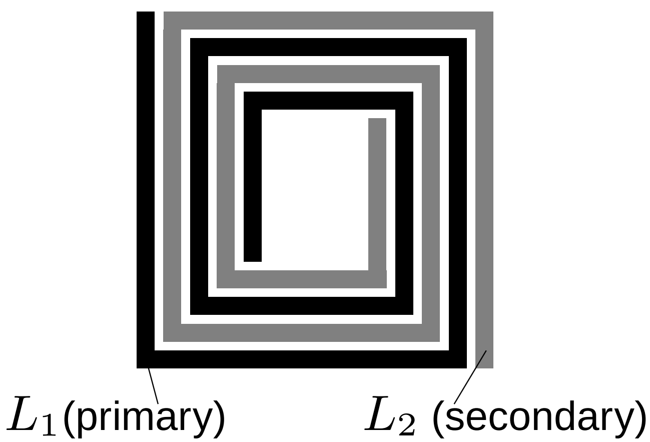
Single inductor.



Tapped transformer.

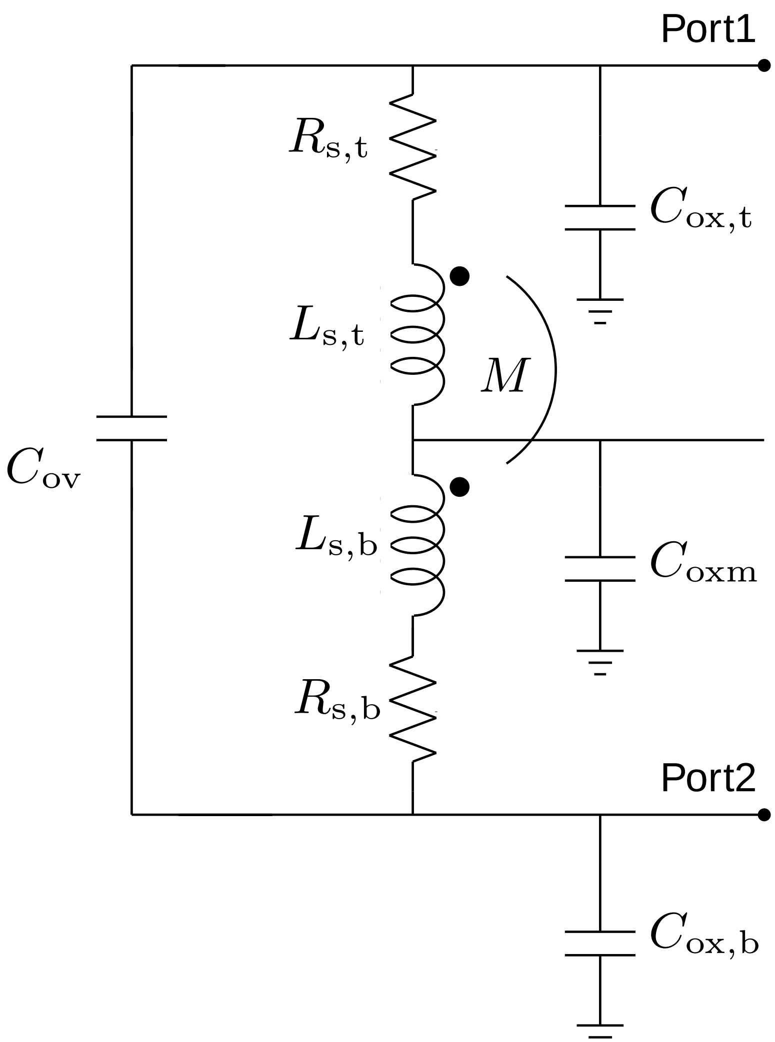


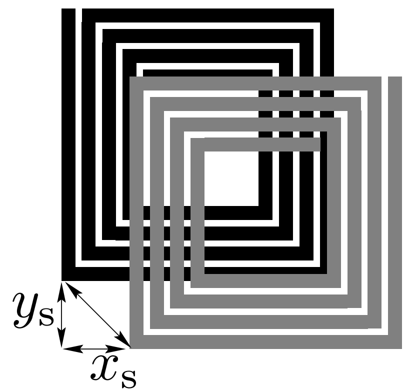
Interleaved transformer.



- $L_T = L_1 + L_2 + 2M$

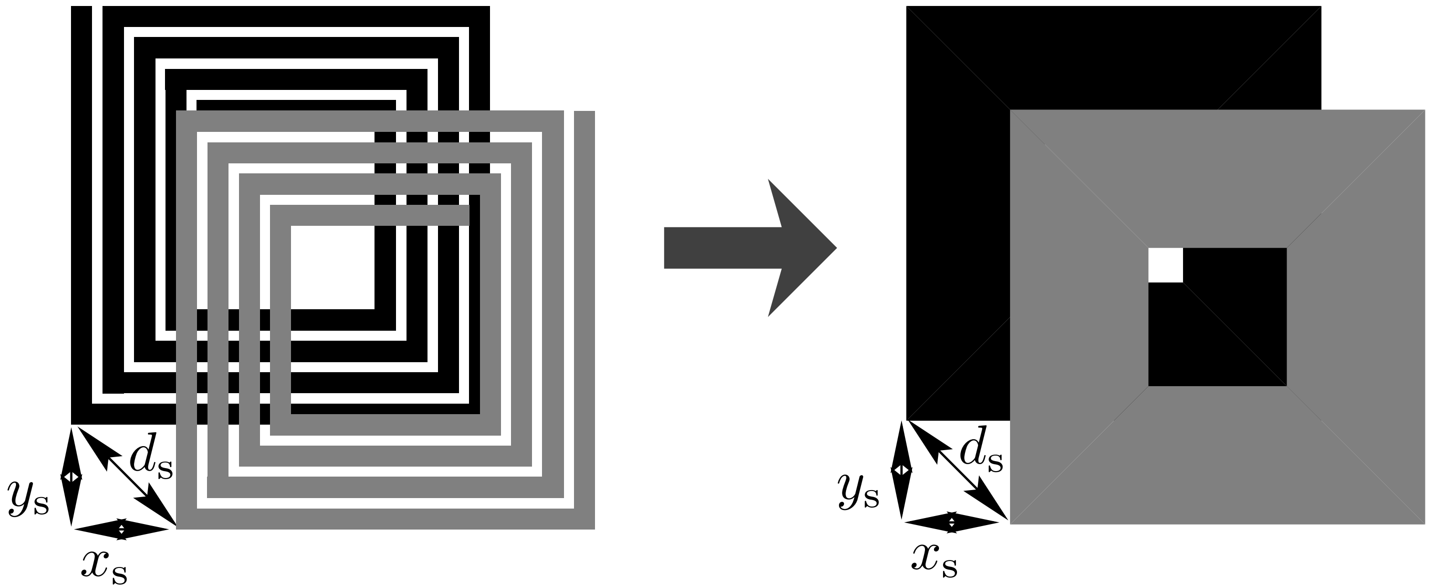
STACKED TRANSFORMER MODEL





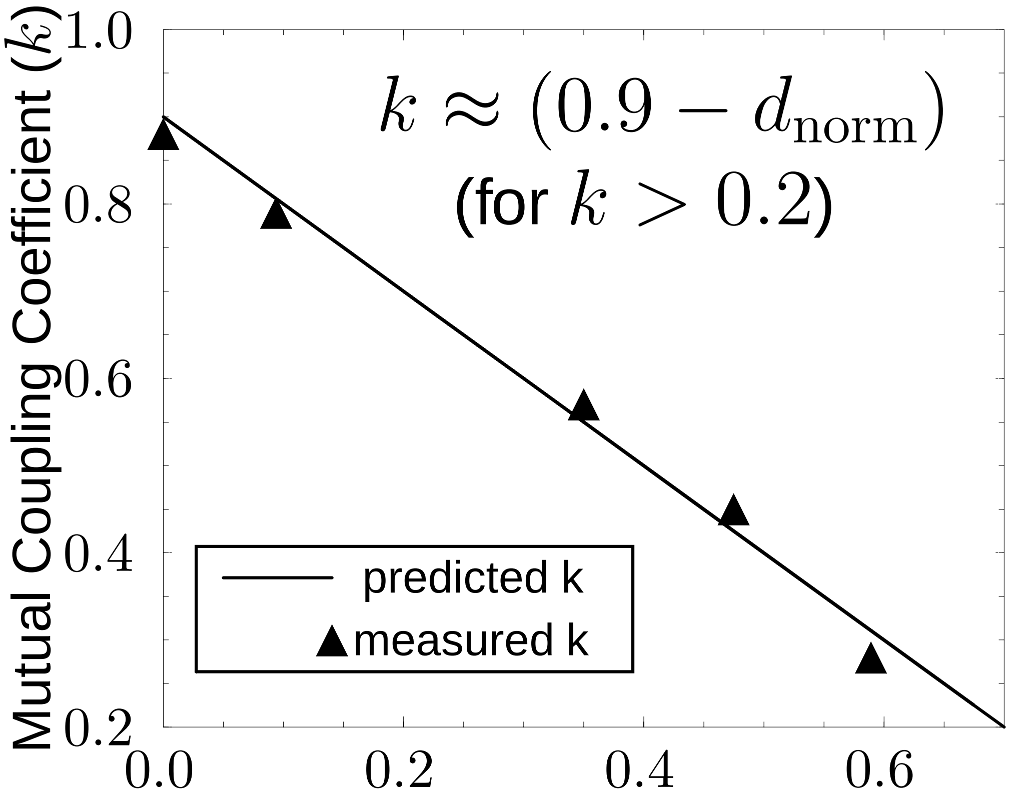
- Evaluate C_{ov}, $C_{ox,t}$, C_{oxm}, $C_{ox,b}$, $R_{s,t}$ & $R_{s,b}$ by extending previous work
- Use inductance expression for $L_{s,t}$, $L_{s,b}$
- Calculate M

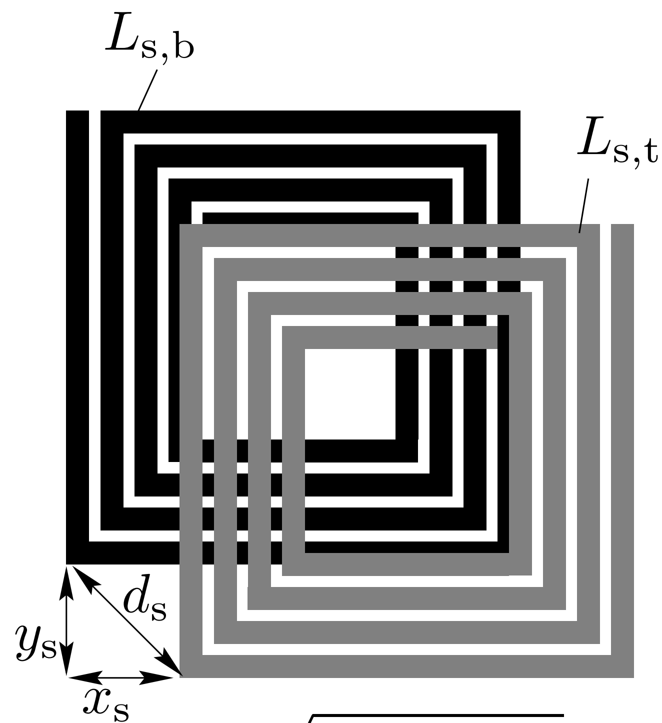
CURRENT SHEET APPROACH FOR k



- Reduce complexity by $4n^2$
- Use symmetry
- Derive simple expression using electromagnetic theory

k FOR STACKED TRANSFORMERS



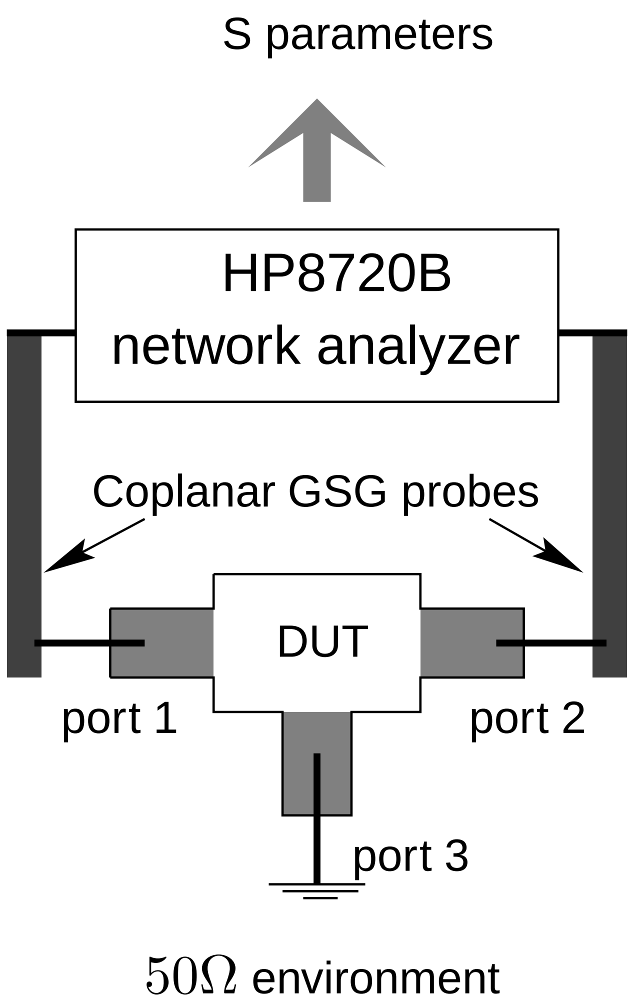


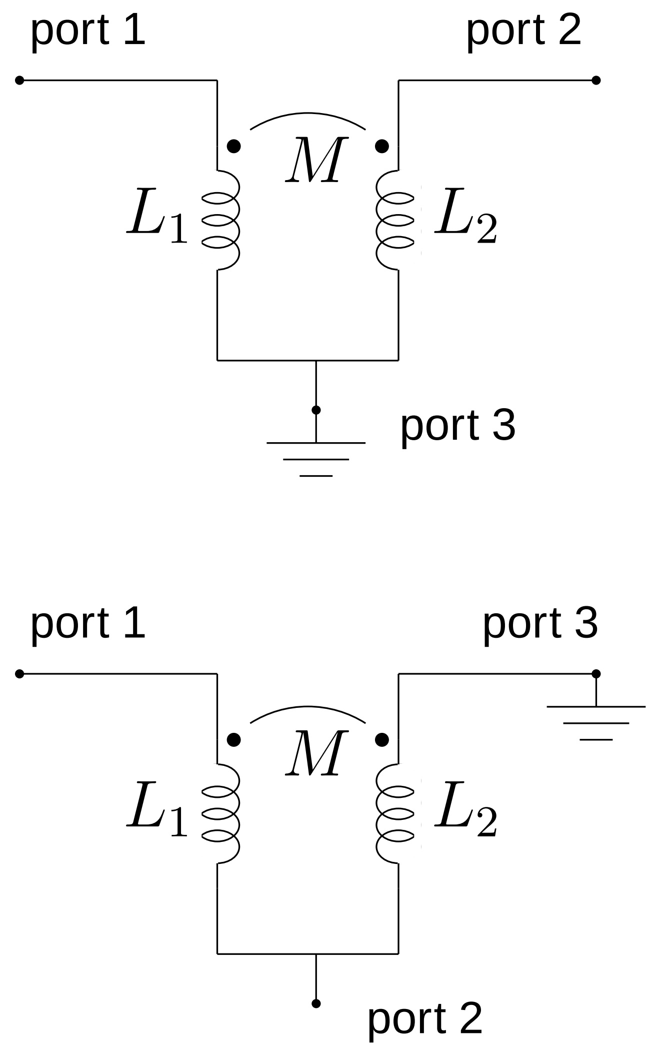
$$M = k\sqrt{(L_{\text{s,t}} L_{\text{s,b}})}$$

$$d_{\text{norm}} = \frac{\sqrt{{x_{\text{s}}}^2 + {y_{\text{s}}}^2}}{\text{AD}} = \frac{d_{\text{s}}}{\text{AD}}$$

- Metal and oxide thicknesses have only 2nd order effects on k

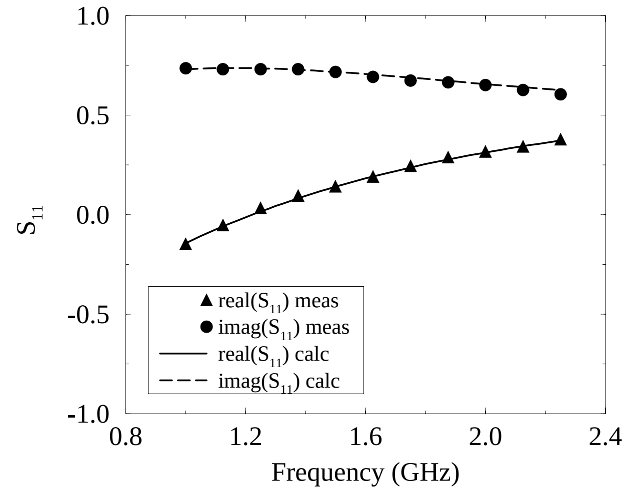
EXPERIMENTAL SET-UP

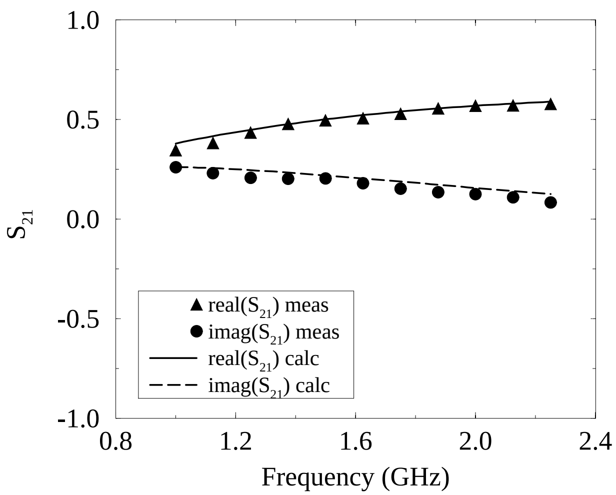


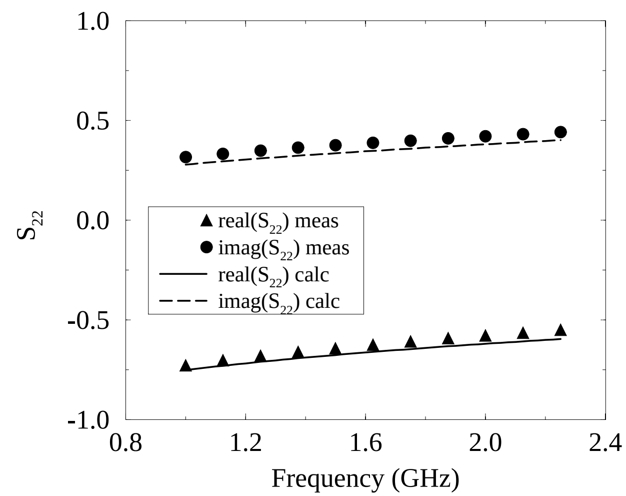


EXPERIMENTAL VERIFICATION: TAPPED

- $\mathrm{OD_o} = 290\mu\mathrm{m}$, $n_\mathrm{o} = 2.5$
- $\mathrm{OD_i} = 190\mu\mathrm{m}$, $n_\mathrm{i} = 4.25$
- $w = 13\mu\mathrm{m}$, $s = 7\mu\mathrm{m}$

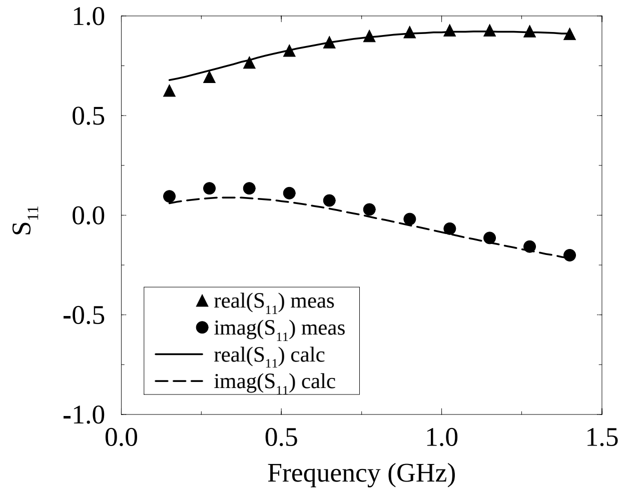


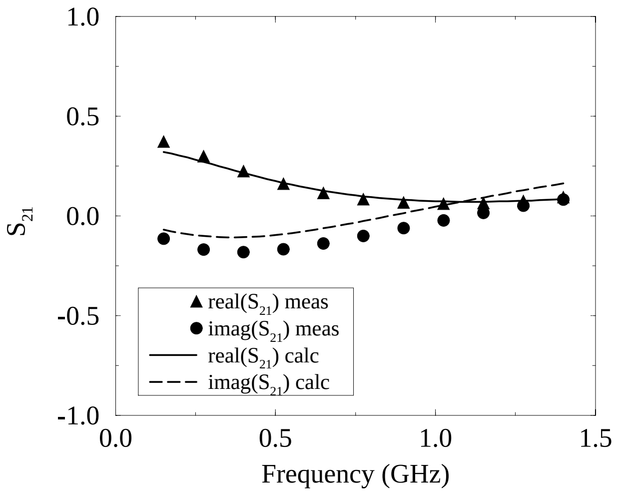


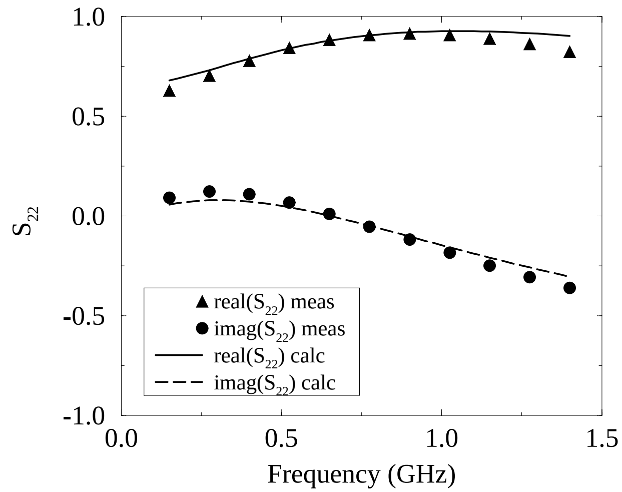


EXPERIMENTAL VERIFICATION: STACKED 1

- Stacked transformer with top spiral overlapping bottom one
- $\text{OD} = 180\mu\text{m}$, $n = 11.75$, $w = 3.2\mu\text{m}$, $s = 2.1\mu\text{m}$
- $x_\text{s} = 0\mu\text{m}$, $y_\text{s} = 0\mu\text{m}$, $d_\text{s} = 0\mu\text{m}$







Future Work

- Incorporate inductive coupling to substrate: significant in CMOS epi processes
- Improve expressions for the series resistance to include proximity effects
- Extend current sheet approach to handle non-uniform current distributions

CONTRIBUTIONS

- Current sheet approach to inductance calculation
- Simple accurate expression for inductance of sdquare, hexagonal, octagonal and circular spirals
- Expressions for mutual inductance and mutual coupling coefficient
- On-chip transformer models
- Basis for design and synthesis of on-chip inductor and transformer circuits
- Shunt-peaked amplifier with optimized on-chip inductor

SO WHAT ?

- Design
  - Scalable, analytical models for synthesis and optimization
- Verification
  - Field solvers