Phase Noise in Oscillators

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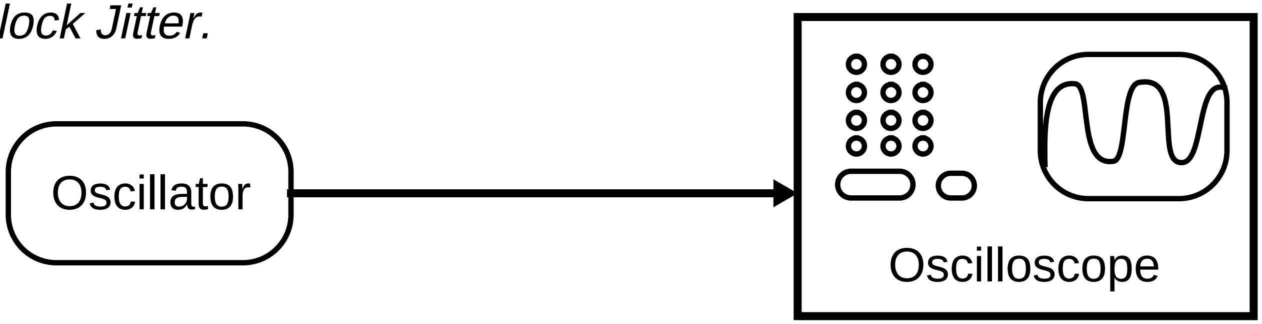
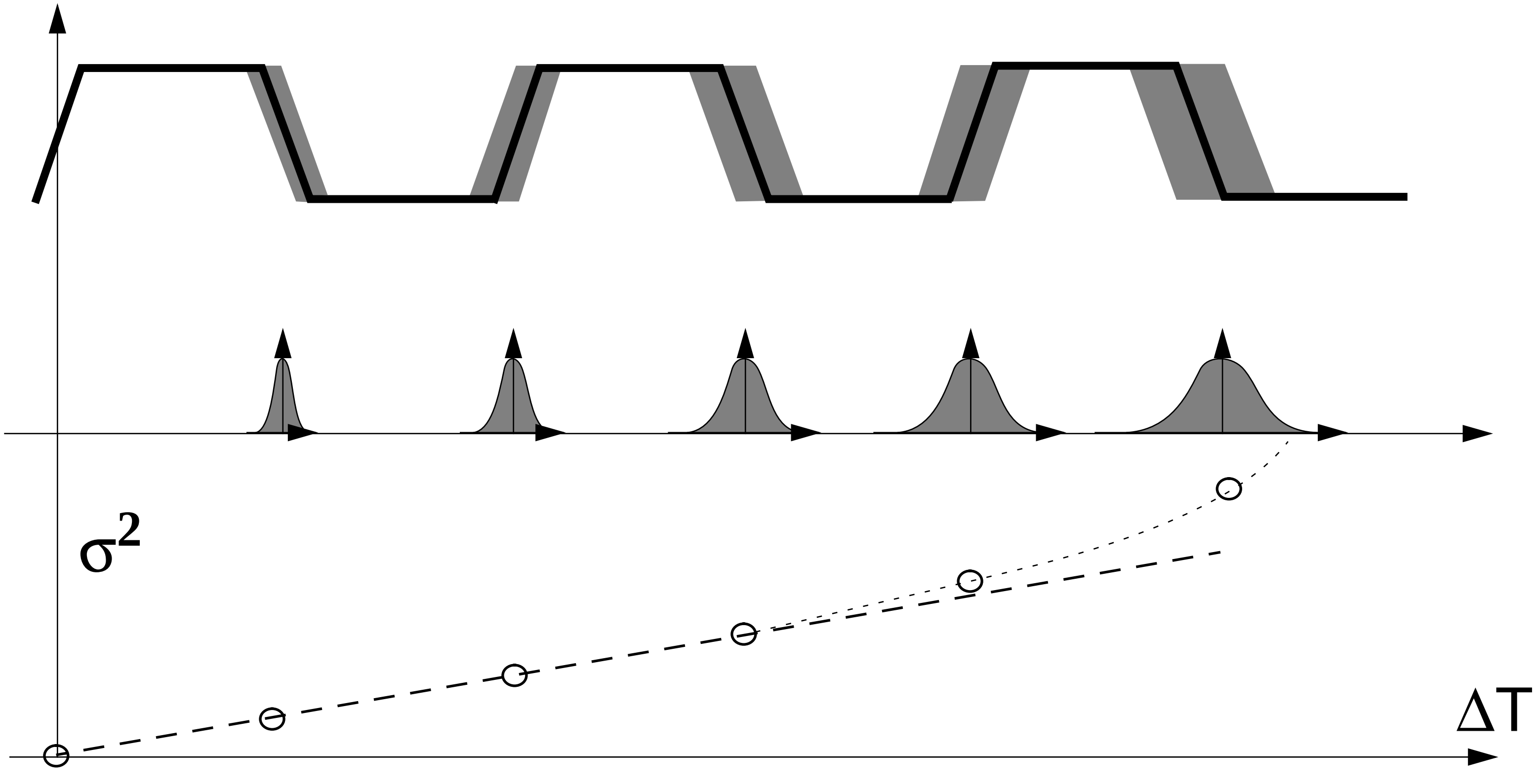
Outline

- ■ ***Introduction and Definitions***
- ☐ Time-Variant Phase Noise Model
  - ○ Upconversion of 1/f Noise
  - ○ Cyclostationary Noise Sources
  - ○ Substrate and Supply Noise
- ☐ Measurement Results
- ☐ Conclusion

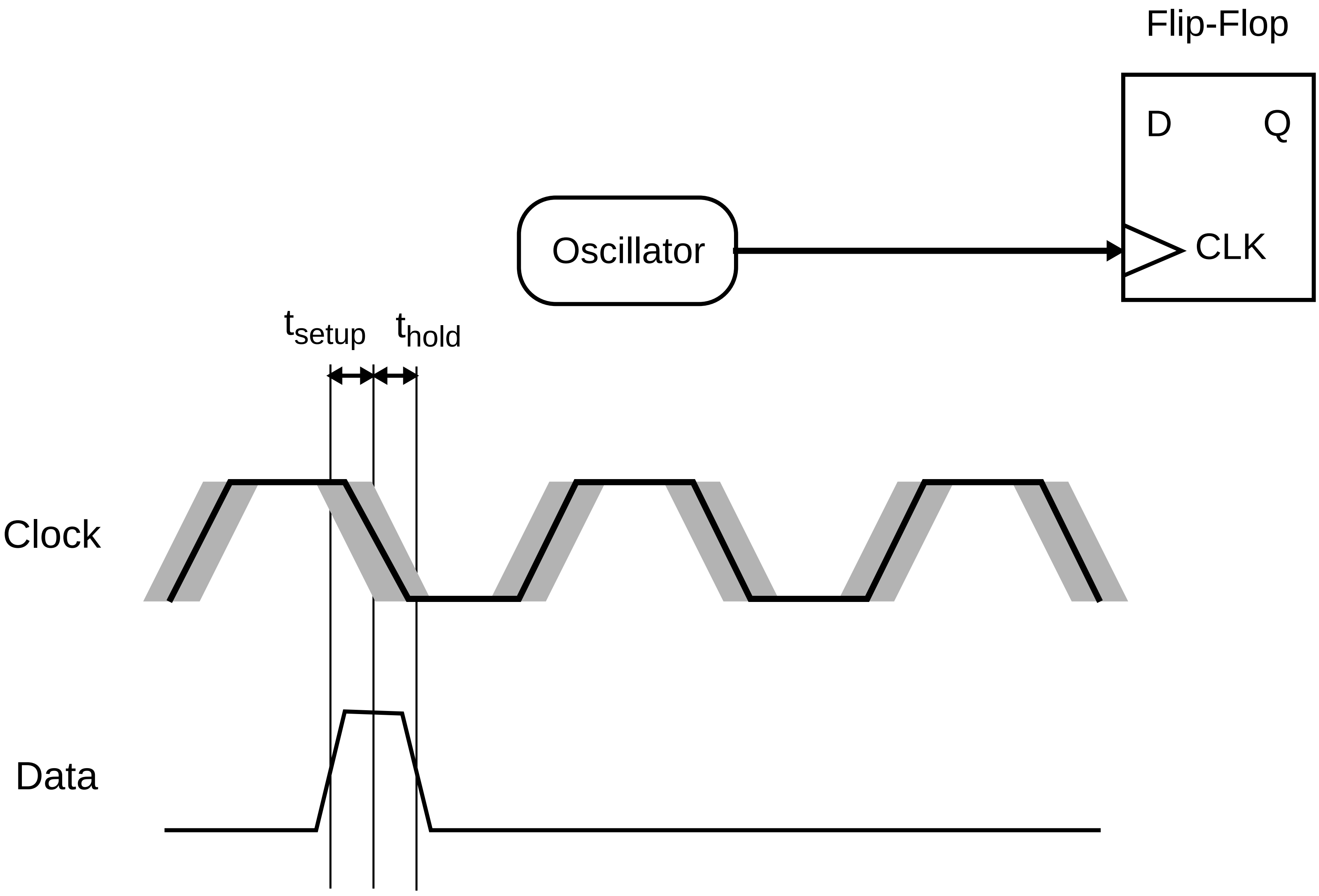
Frequency Instability: Time Domain

Known As Clock Jitter.



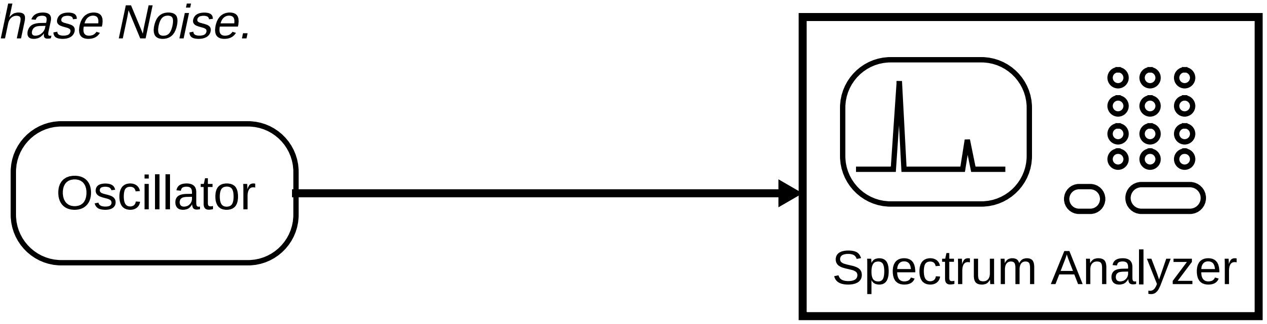


Timing Jitter in Digital Applications



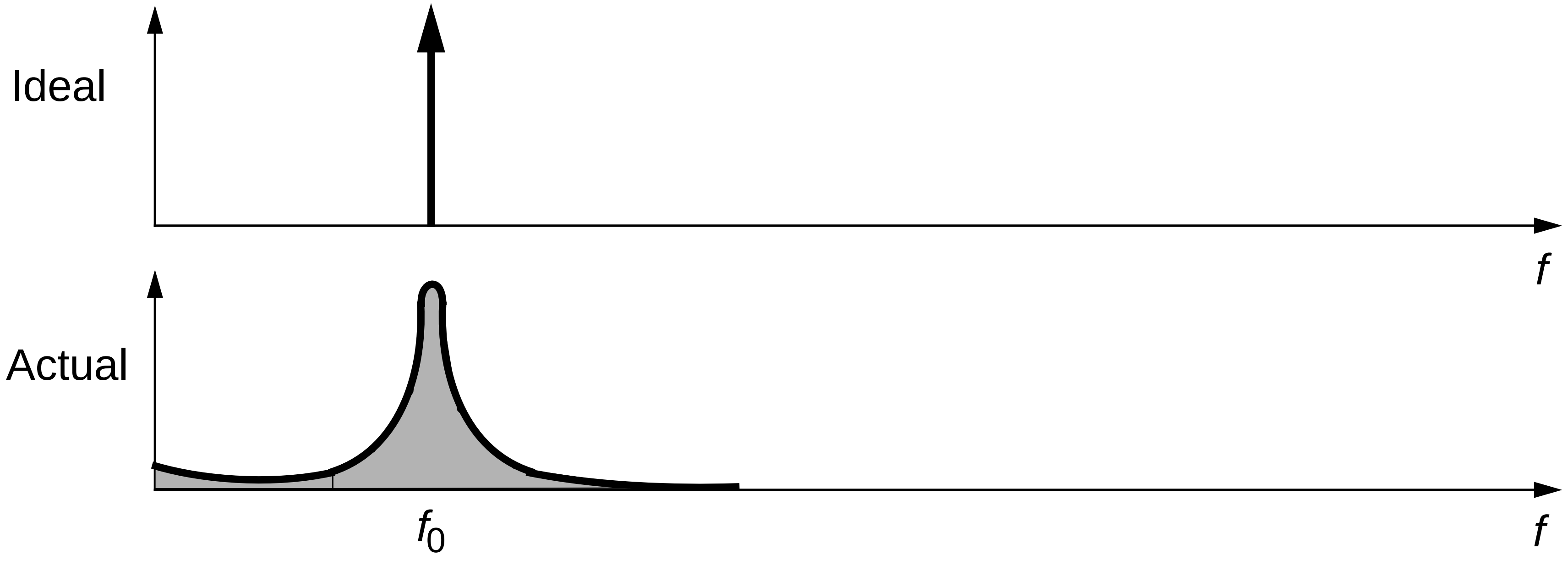
Frequency Instability: Frequency Domain

Known As Phase Noise.

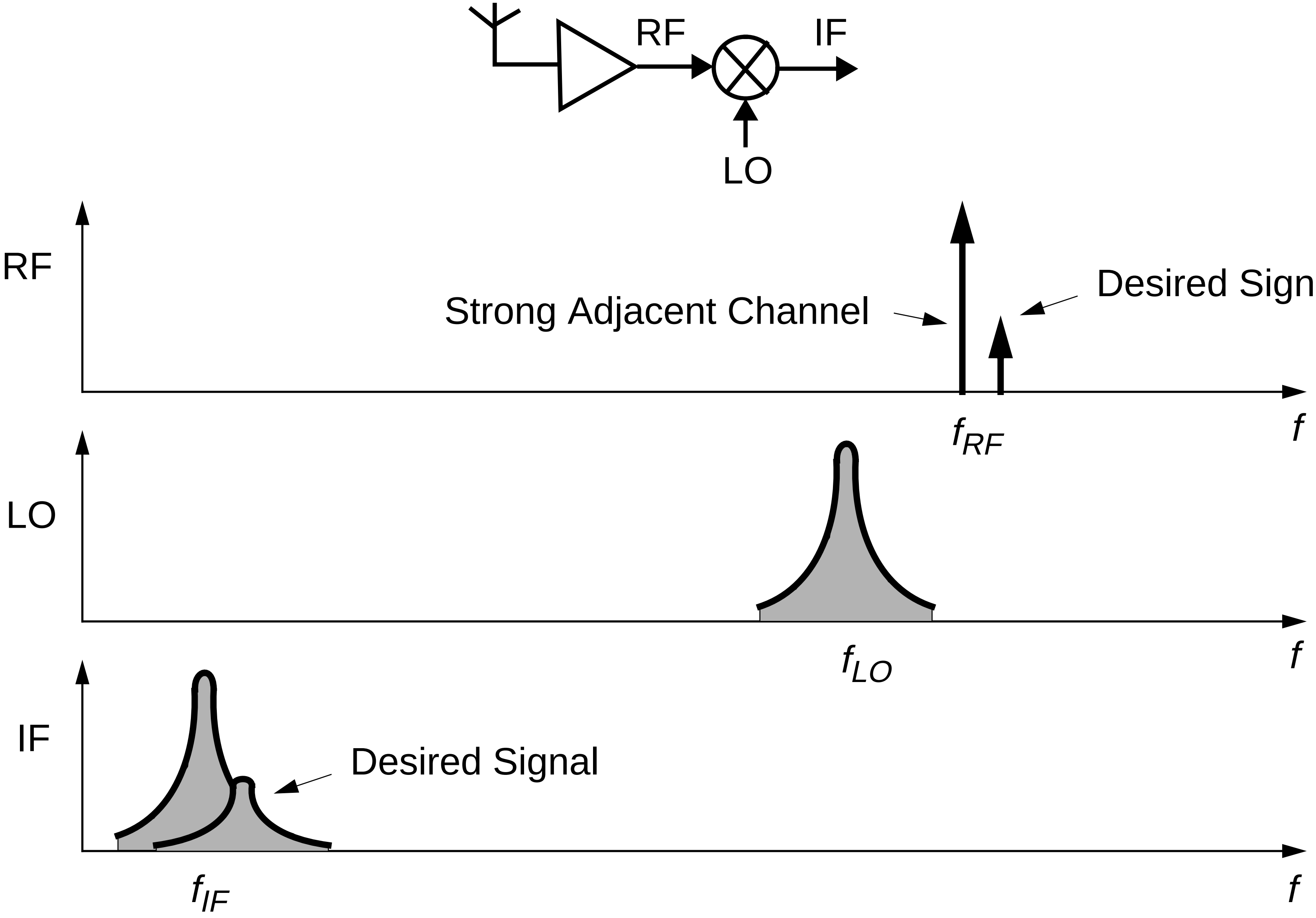


Oscillator

Spectrum Analyzer

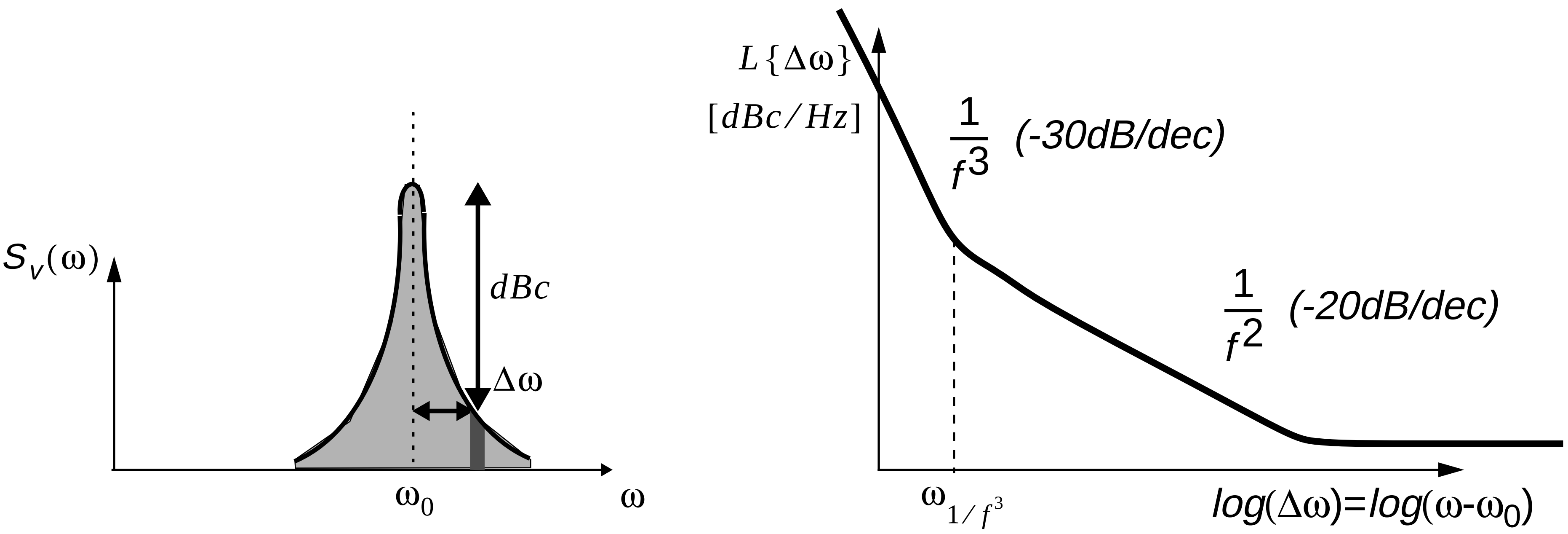


Phase Noise in RF Applications



The desired signal is buried under the phase noise of an adjacent strong channel.

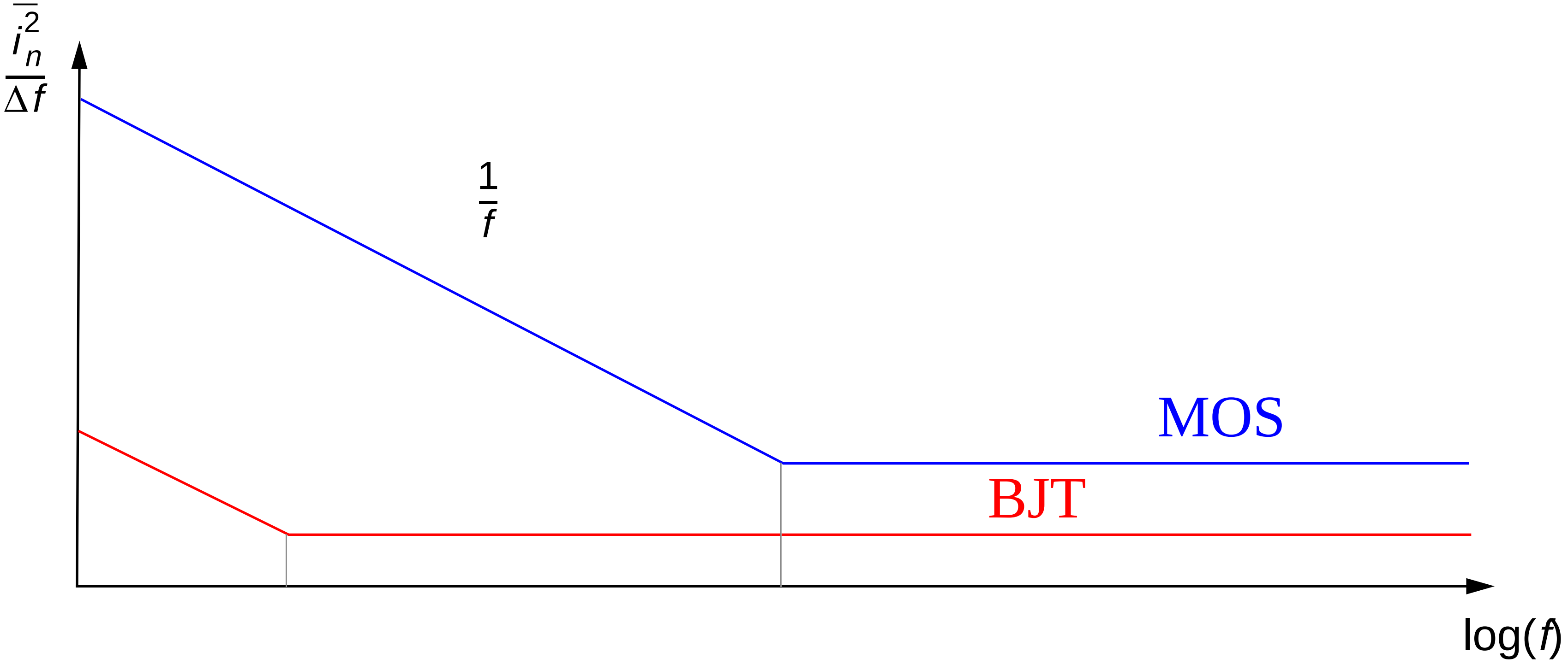
Units of Phase Noise



Measured in dB below carrier per unit bandwidth.

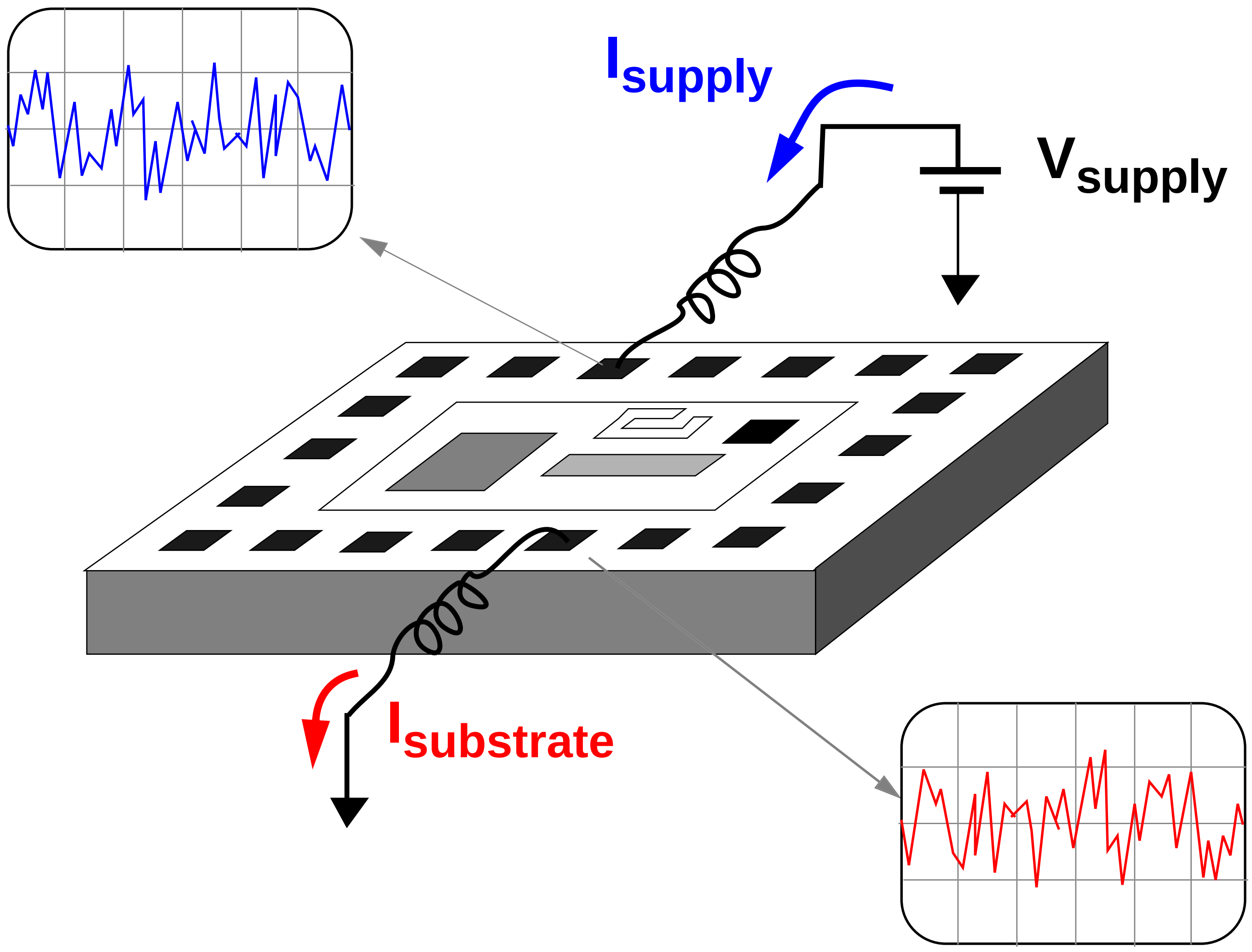
Thermal and 1/*f* Noise

Internal noise sources set a fundamental limit for phase noise.



Low frequency noise can be an important contributor to the system noise.

Substrate and Supply Noise



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■ ***Time-Variant Phase Noise Model***

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  ○ Substrate and Supply Noise

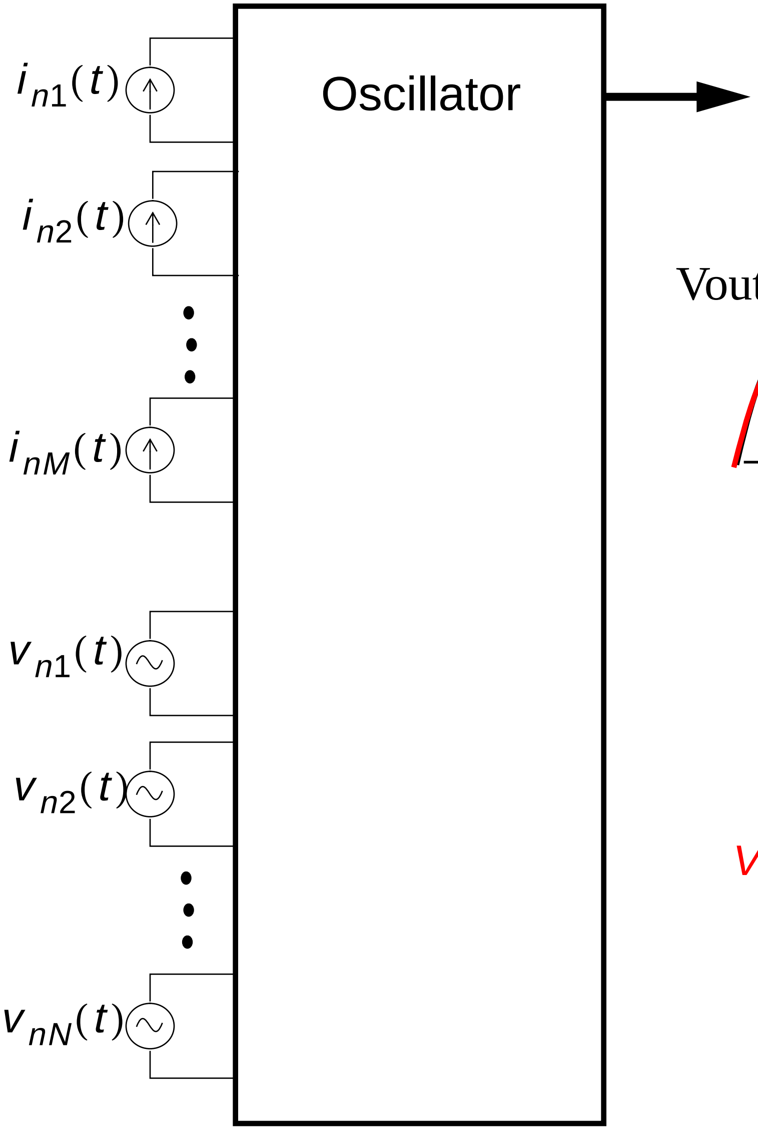
☐ Measurement Results

☐ Conclusion

Oscillator with Input Noise Sources

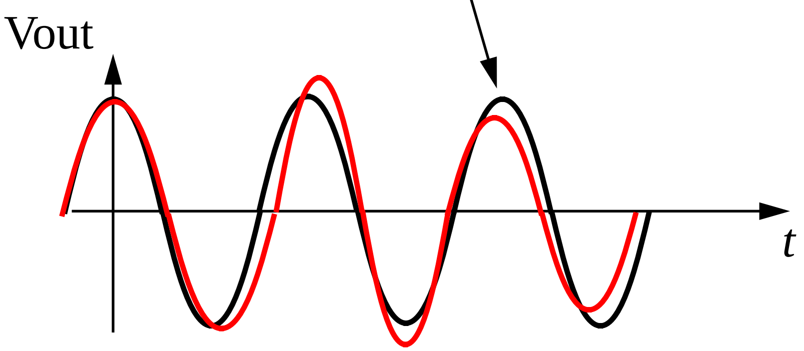
Noise current sources.

Noise voltage sources.



Ideal waveform

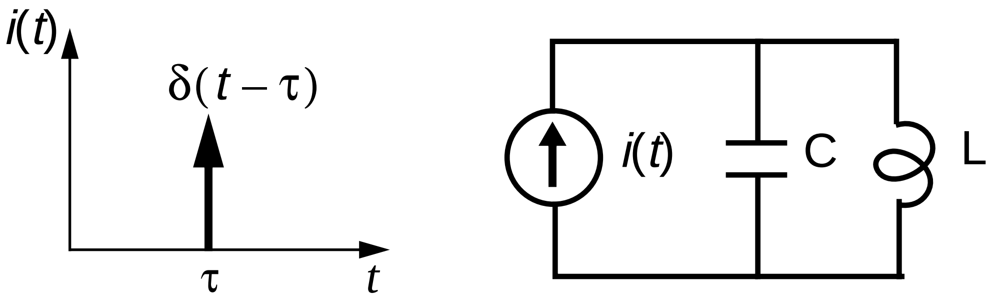
$$V_{out}(t) = A \cdot \cos(\omega_0 t + \phi_0)$$

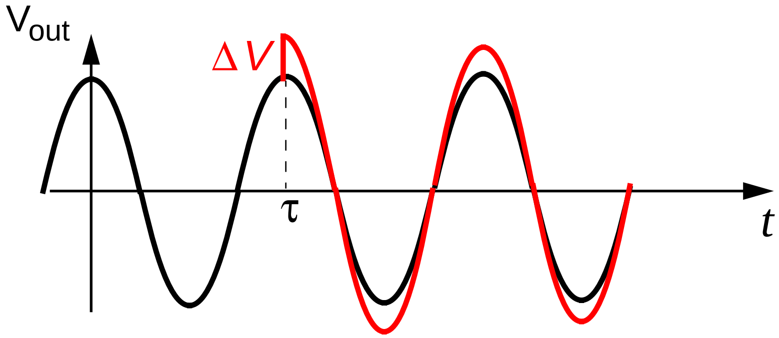


Non-ideal waveform

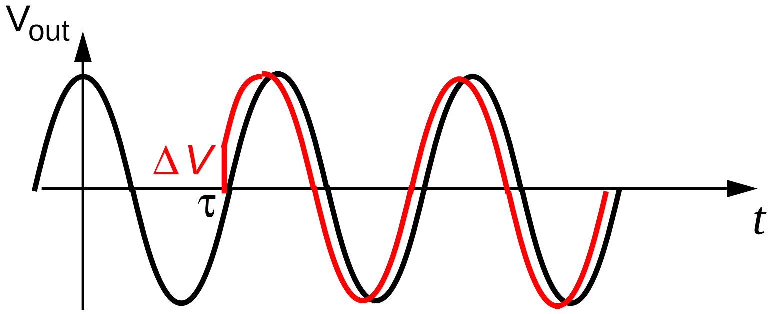
$$V_{out}(t) = A(t) \cdot f[\omega_0 t + \phi(t)]$$

Oscillators Are Time-Variant Systems





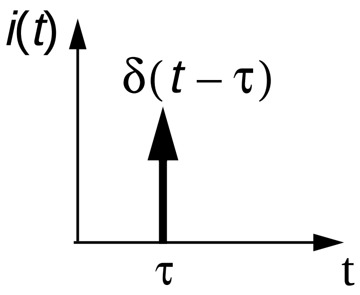
Impulse injected at the peak of amplitude.

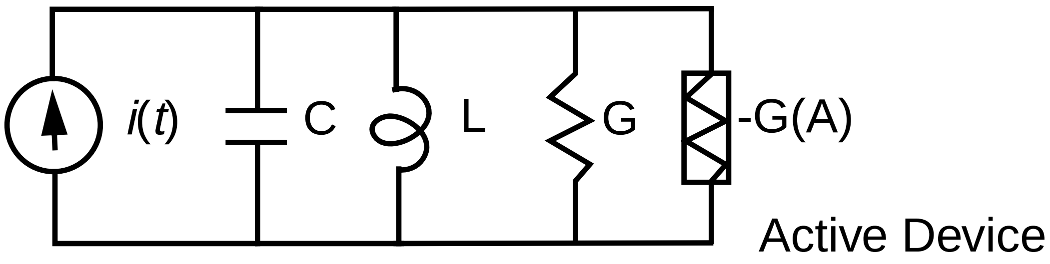


Impulse injected at zero crossing.

Even for an ideal LC oscillator, the phase response is *Time Variant.*

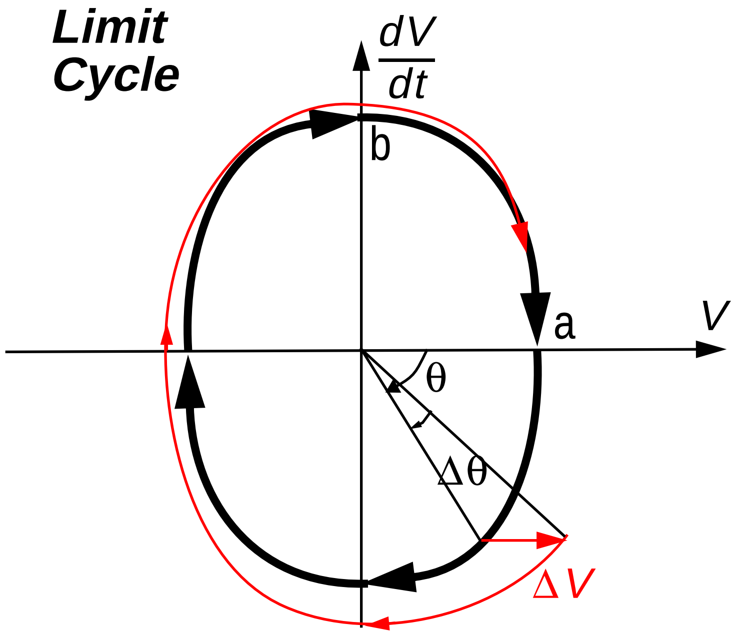
Amplitude Restoring Mechanism



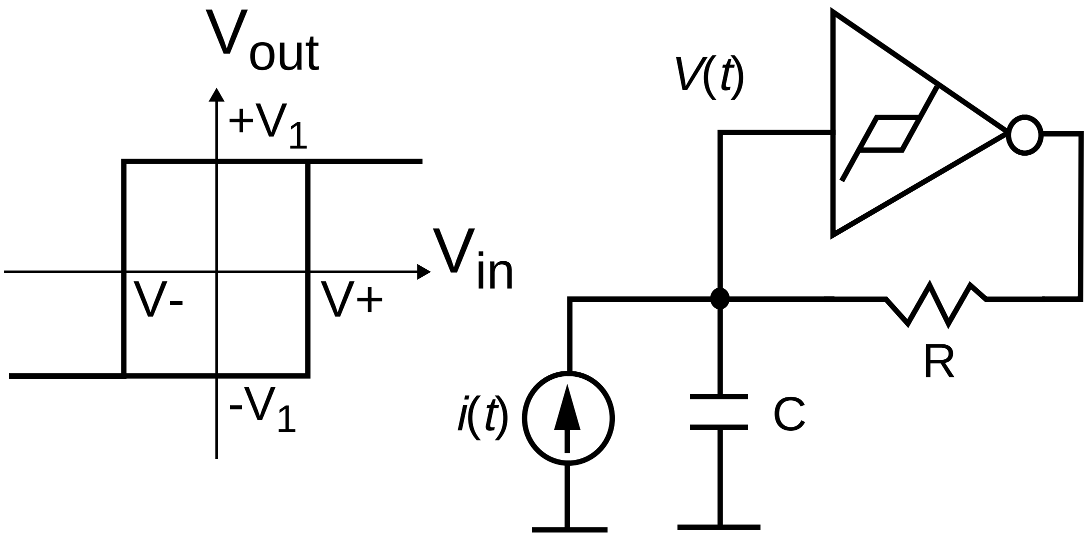


Once Introduced, phase error persists indefinitely.

Non-linearity quenches amplitude changes over time.



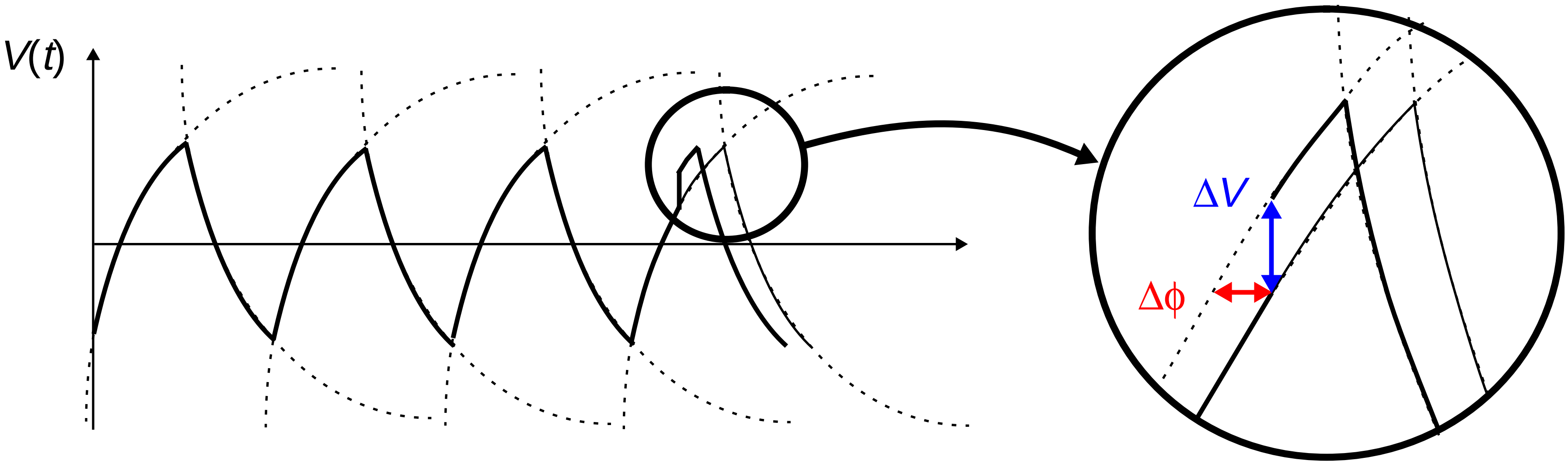
Impulse Response of a Relaxation Oscillator



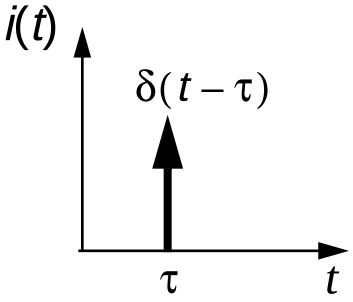
$$\Delta V = \frac{\Delta q}{C_{node}}$$

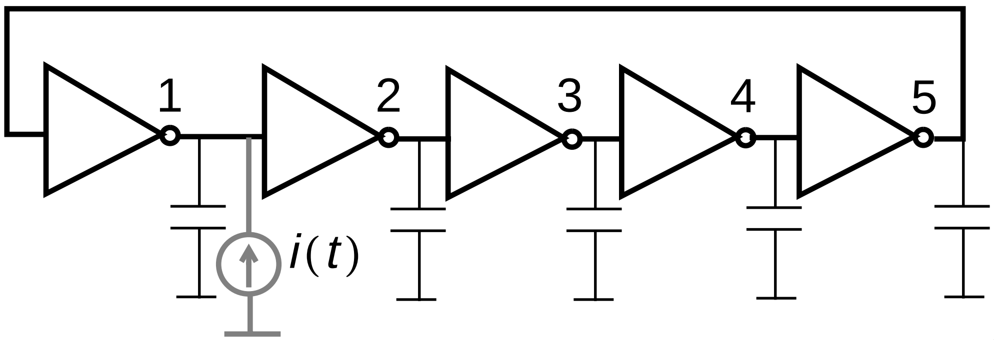
$$\Delta\phi = \Gamma(\omega_0 t)\frac{\Delta V}{V_{swing}} = \Gamma(\omega_0 t)\frac{\Delta q}{q_{swing}}$$

$$\Delta q \ll q_{swing}$$



Impulse Response of a Ring Oscillator





$$\Delta V = \frac{\Delta q}{C_{node}}$$

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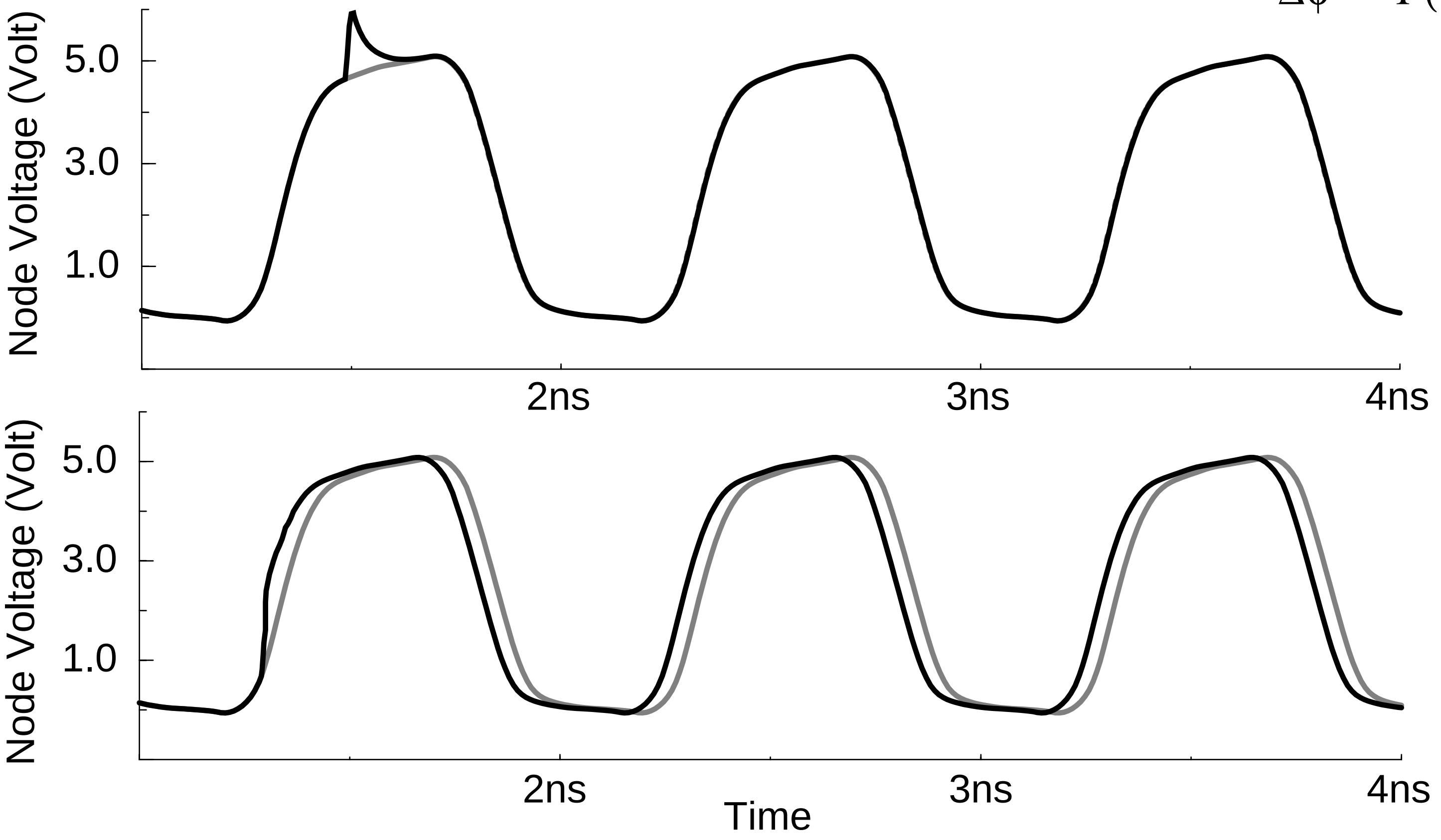
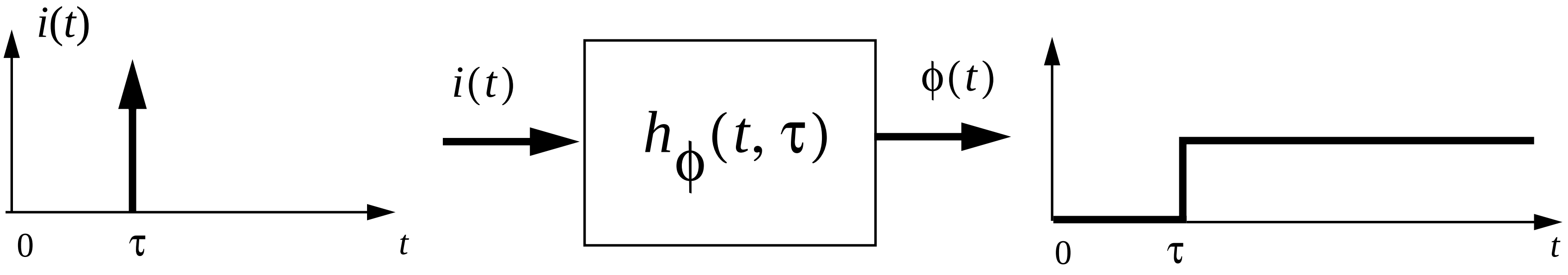


Phase Impulse Response

The phase impulse response of an arbitrary oscillator is a time varying step.

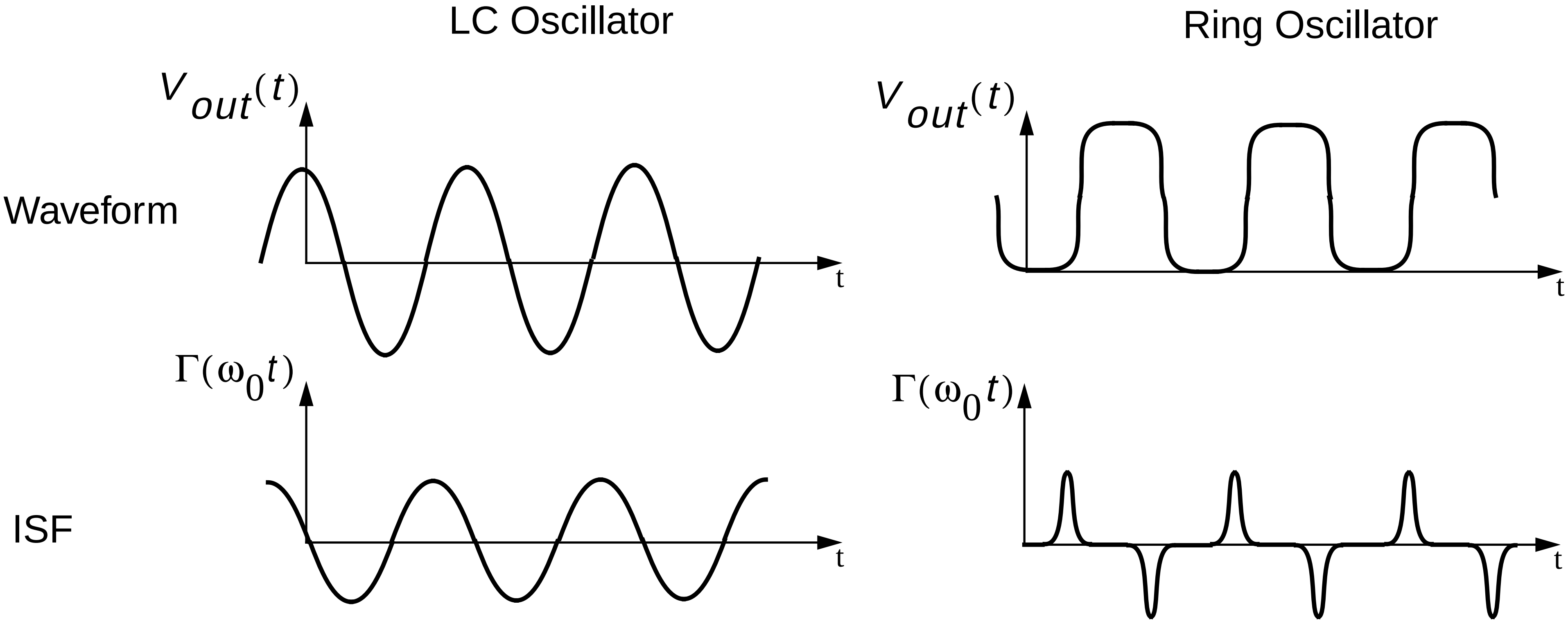


The unit impulse response is:

$$h_\phi(t, \tau) = \frac{\Gamma(\omega_o \tau)}{q_{max}} u(t - \tau)$$

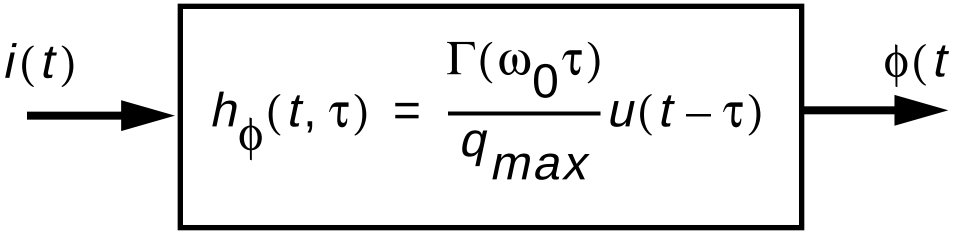
$\Gamma(x)$ is a dimensionless function periodic in 2π, describing how much phase change results from applying an impulse at time: $t = T\frac{x}{2\pi}$

Impulse Sensitivity Function (ISF)



The ISF quantifies the sensitivity of every point in the waveform to perturbations.

Phase Response to an Arbitrary Source



$i(t)$

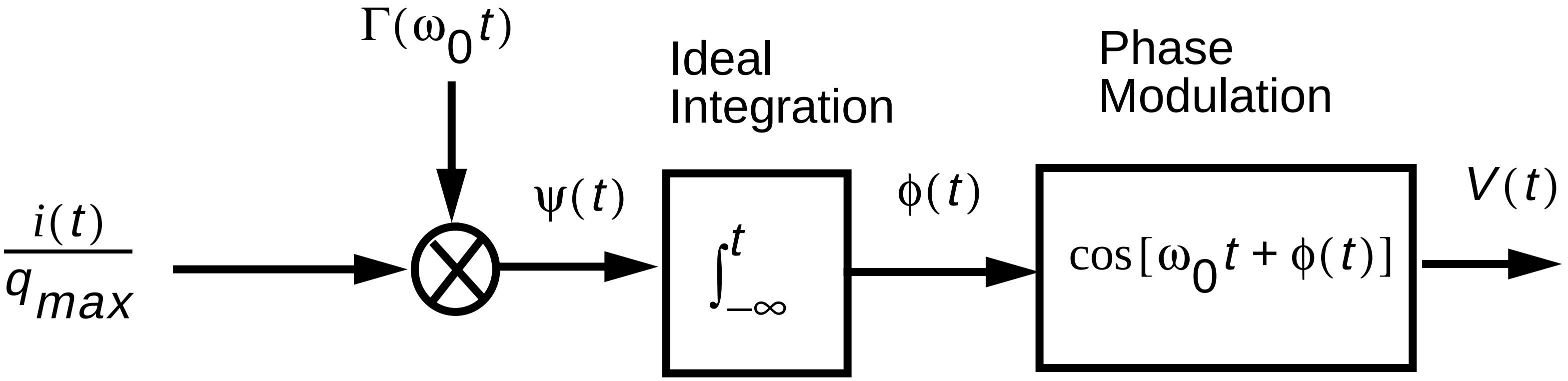
$$h_\phi(t,\tau) = \frac{\Gamma(\omega_0\tau)}{q_{max}}u(t-\tau)$$

$\phi(t)$

Superposition Integral:

$$\phi(t) = \int_{-\infty}^{\infty} h_\phi(t,\tau)i(\tau)d\tau = \frac{1}{q_{max}}\int_{-\infty}^{t}\Gamma(\omega_0\tau)i(\tau)d\tau$$

Equivalent representation:



$\Gamma(\omega_0 t)$

$\frac{i(t)}{q_{max}}$

$\psi(t)$

Ideal Integration

$\int_{-\infty}^{t}$

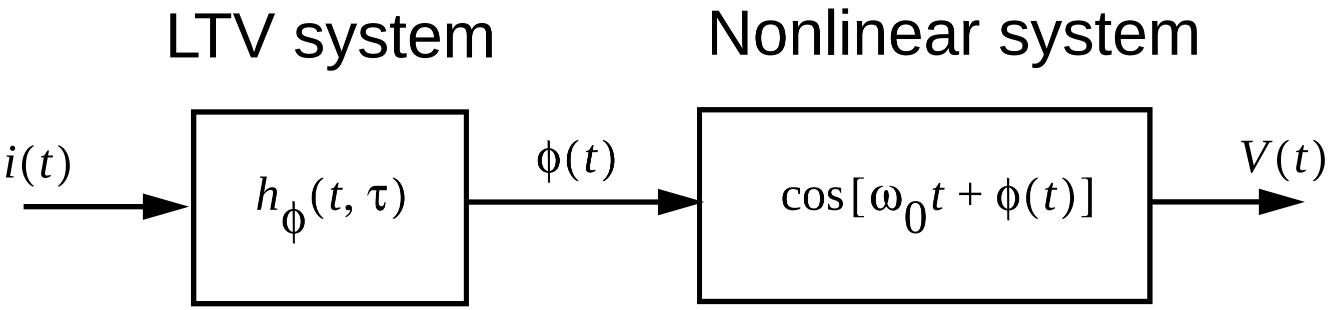
$\phi(t)$

Phase Modulation

$\cos[\omega_0 t + \phi(t)]$

$V(t)$

Phase Noise Due to White Noise



For a white input noise current with the spectral density of $\frac{\overline{i_n^2}}{\Delta f}$

The phase noise sideband power below carrier at an offset of $\Delta\omega$ *is:*

$$L\{\Delta\omega\} = \frac{\Gamma_{rms}^2}{q_{max}^2} \cdot \frac{\overline{i_n^2}/\Delta f}{2\Delta\omega^2}$$

Γ_{rms} *is the rms value of the ISF.*

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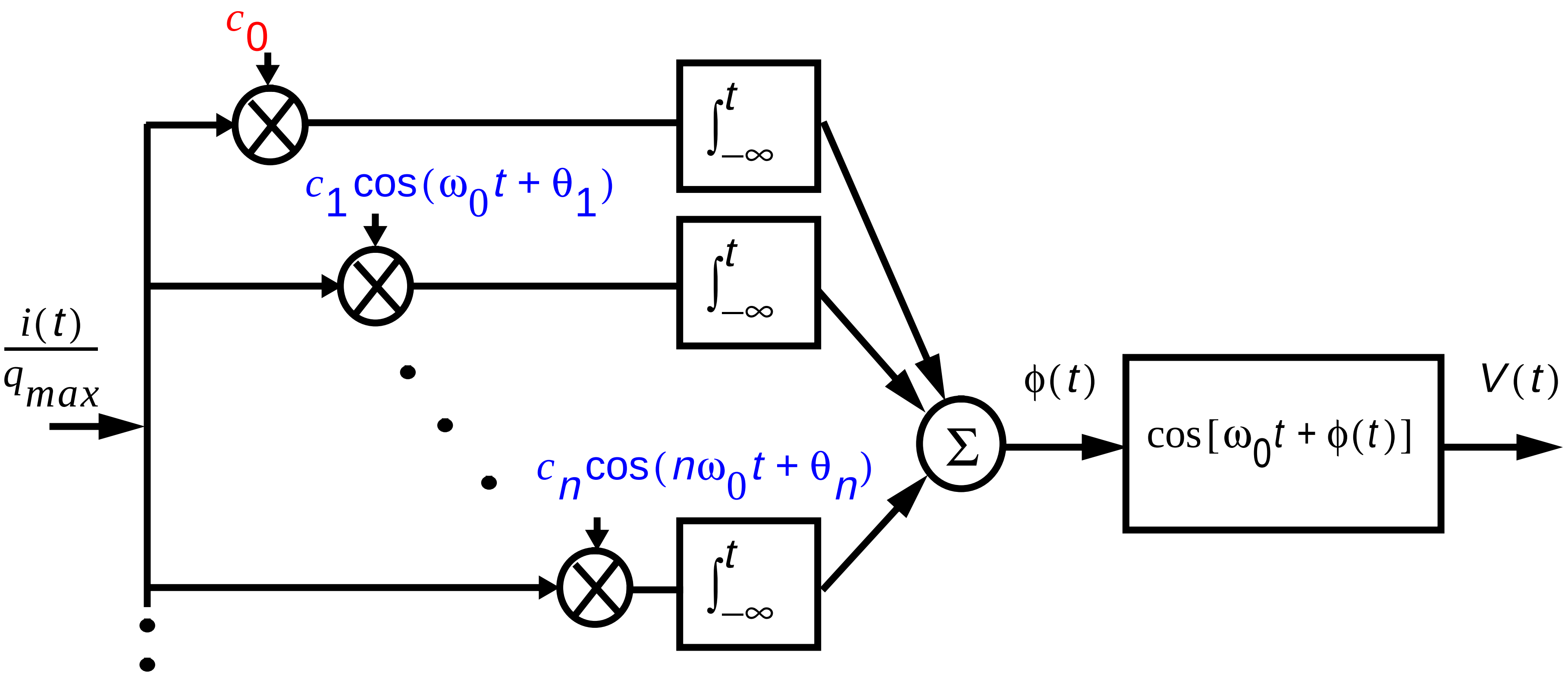
ISF Decomposition

ISF is a periodic function:

$$\Gamma(\omega_o t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0\tau + \theta_n)$$

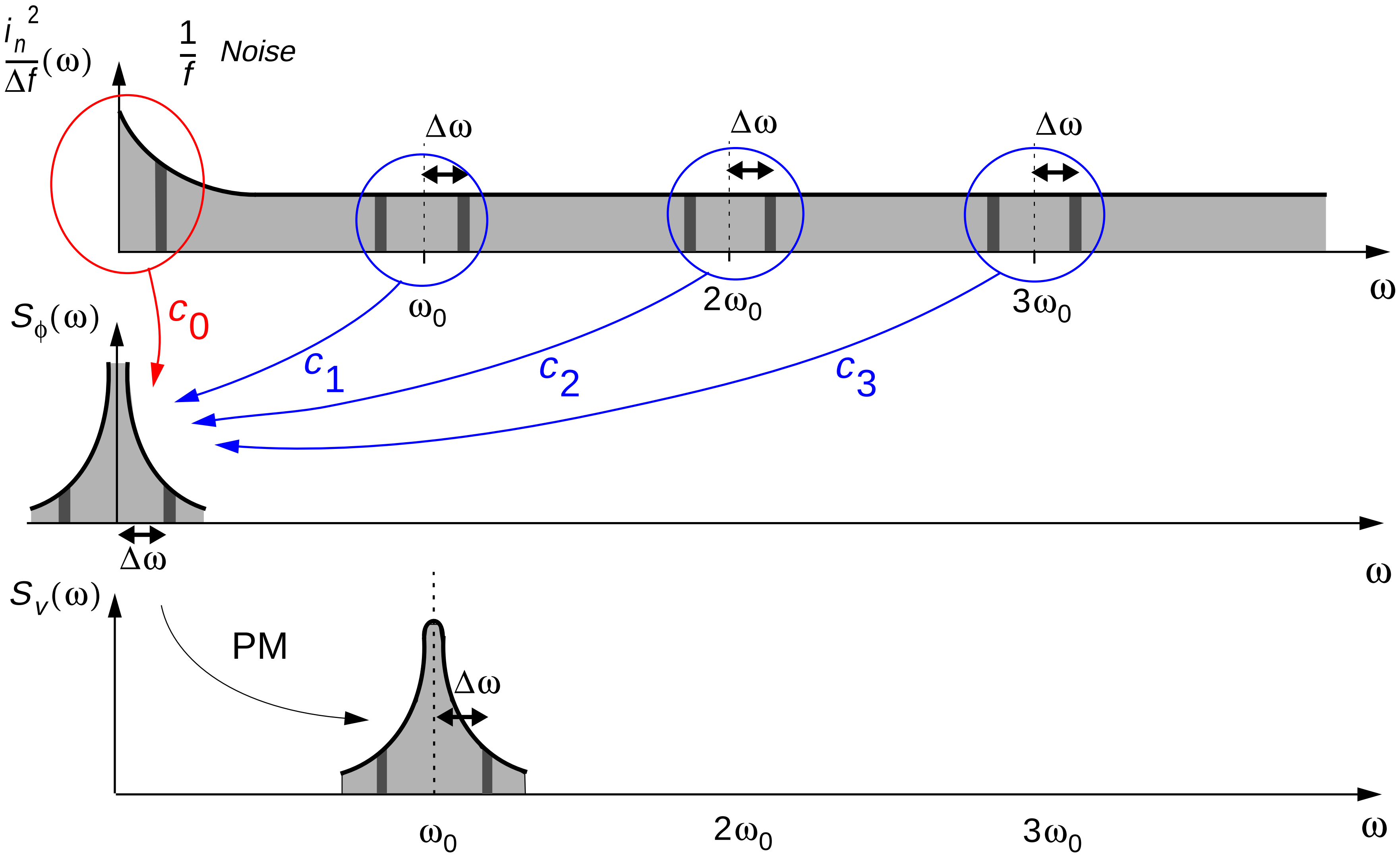
Phase can be written as:

$$\phi(t) = \frac{1}{q_{max}}\left[c_0\int_{-\infty}^{t} i(\tau)d\tau + \sum_{n=1}^{\infty} c_n\int_{-\infty}^{t} i(\tau)\cos(n\omega_0\tau)d\tau\right]$$

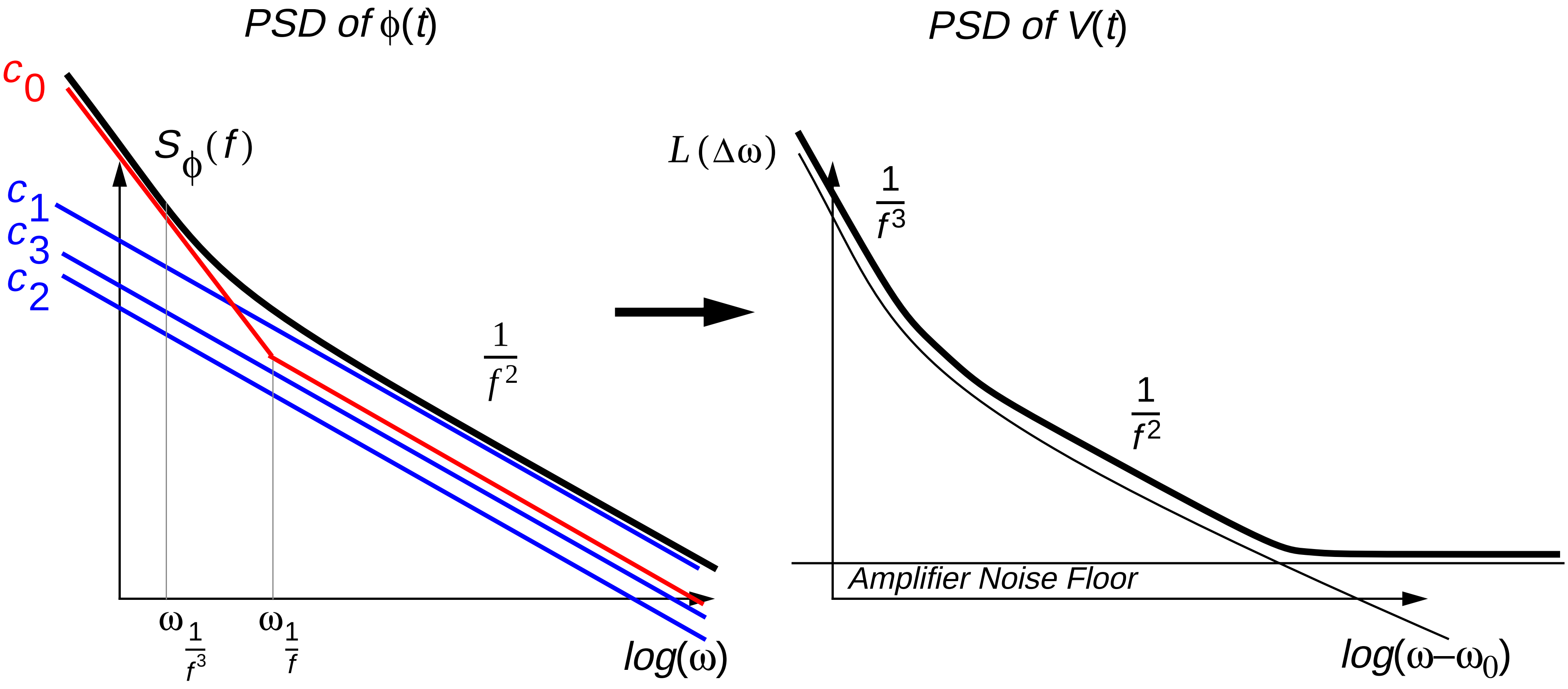


Noise Contributions from $n\omega_o$

$$\phi(t) = \frac{1}{q_{max}}\left[c_0\int_{-\infty}^{t} i(\tau)d\tau + \sum_{n=1}^{\infty} c_n\int_{-\infty}^{t} i(\tau)\cos(n\omega\tau)d\tau\right]$$



Power Spectrum of Phase Noise



Noise components around integer multiples of the oscillation frequency have the strongest effect on phase noise, and their effect is weighted by the Fourier coefficients of the ISF, c_n.

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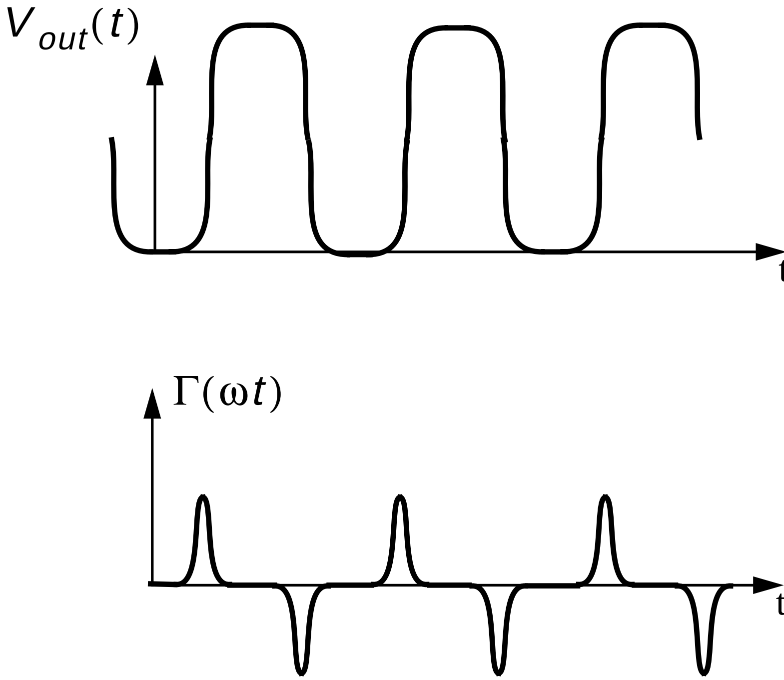
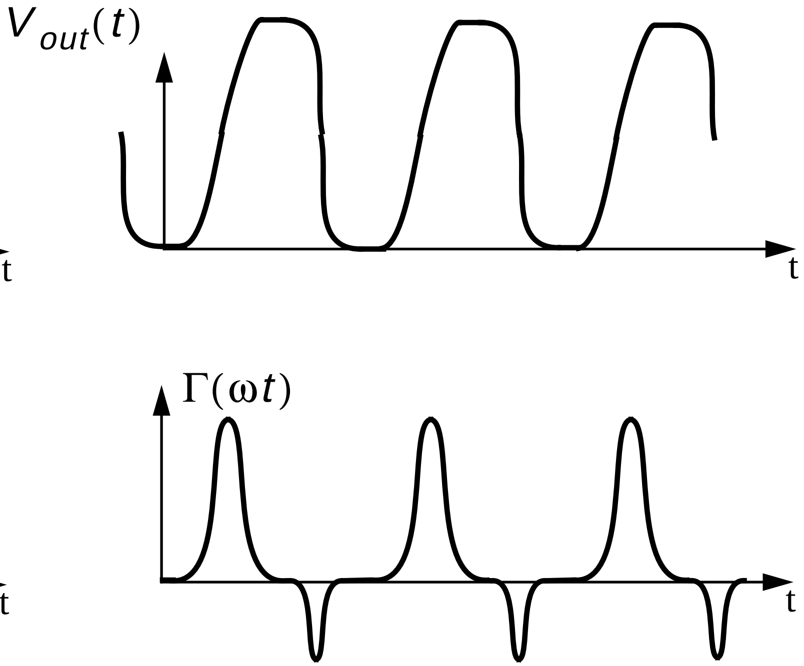
Effect of Symmetry

$$c_0 = \frac{1}{2\pi}\int_0^{2\pi}\Gamma(x)dx$$

Symmetric rise and fall time



Asymmetric rise and fall time



The dc value of the ISF is governed by rise and fall time symmetry, and controls the contribution of low frequency noise to the phase noise.

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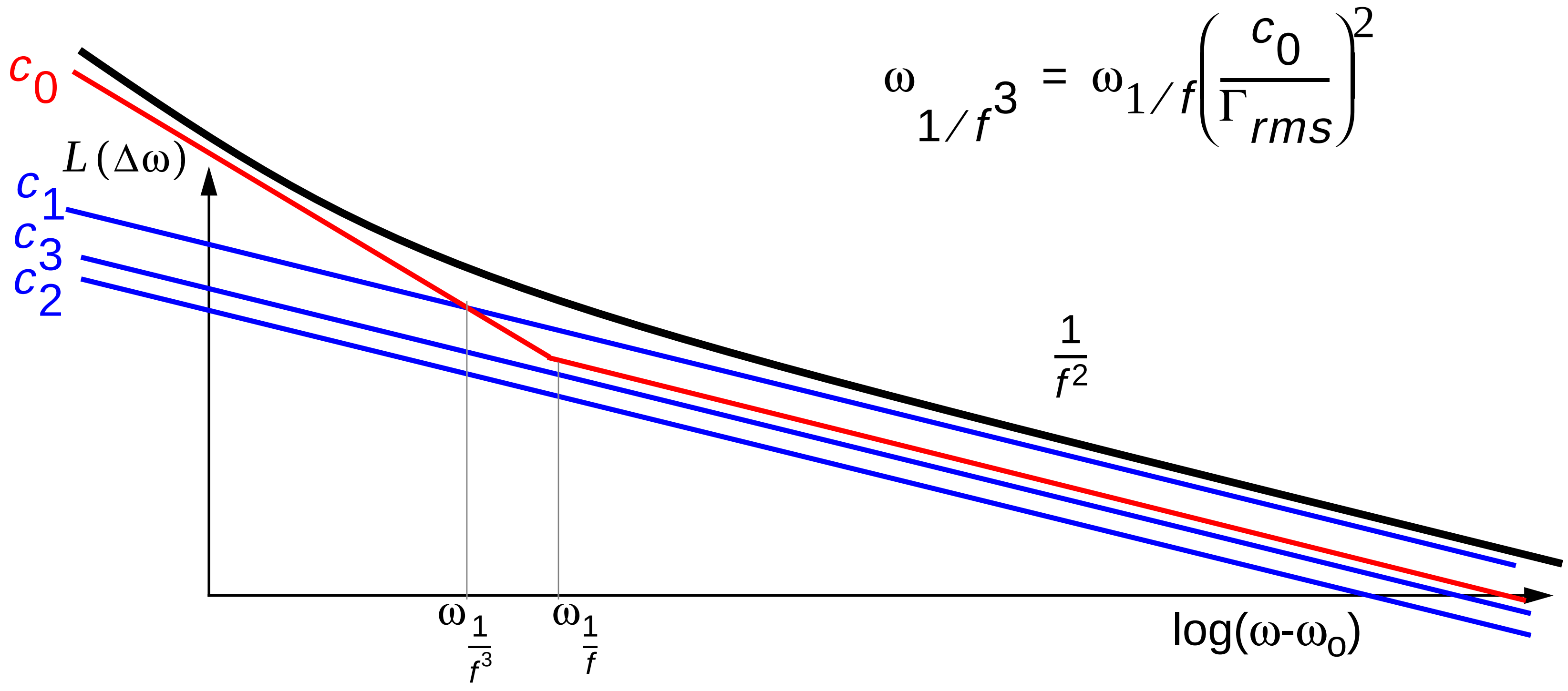
$1/f^3$ Corner of Phase Noise Spectrum

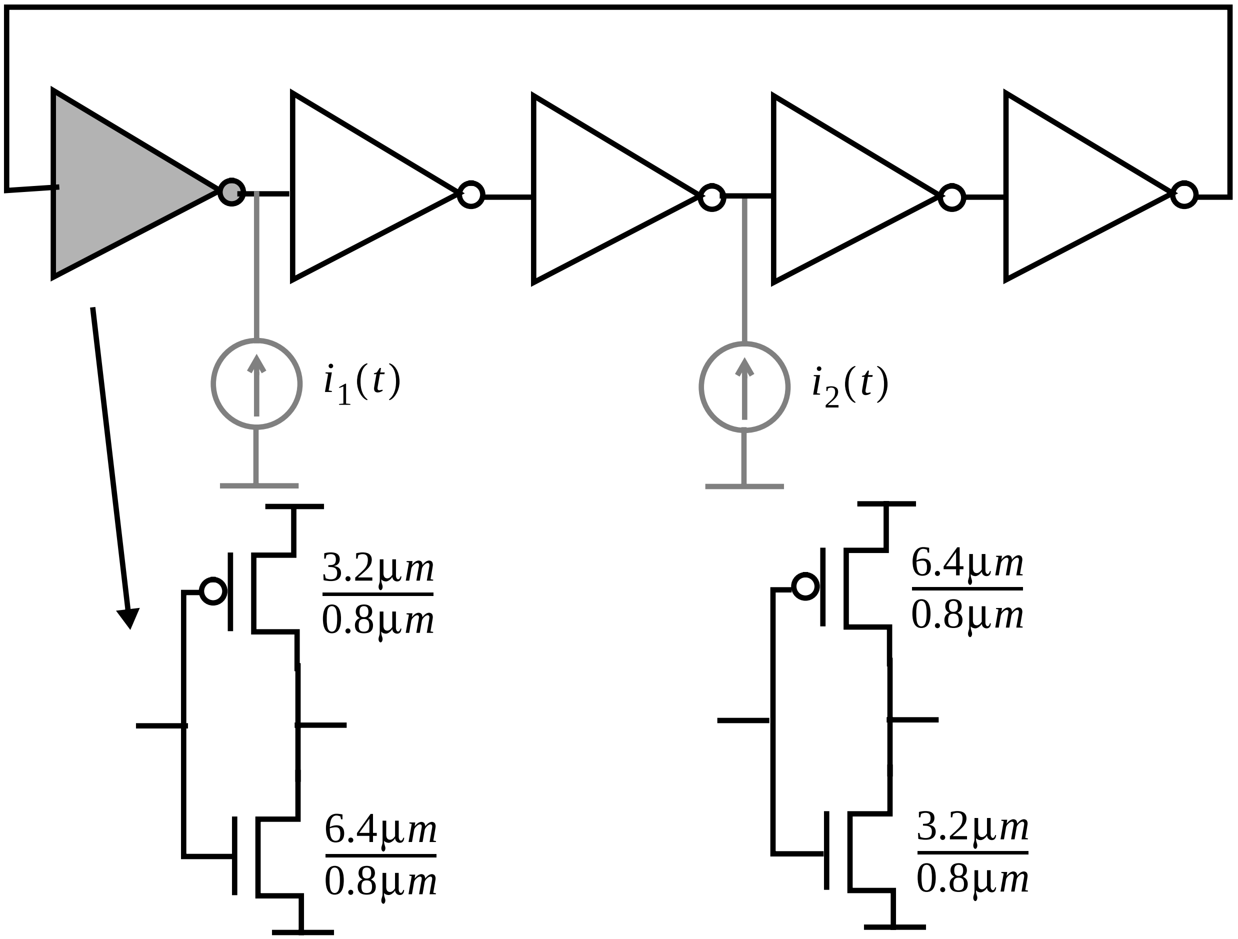
The $1/f^3$ corner of phase noise is NOT the same as 1/f corner of device noise



$$\omega_{1/f^3} = \omega_{1/f}\left(\frac{c_0}{\Gamma_{rms}}\right)^2$$

By designing for a symmetric waveform, the performance degradation due to low frequency noise can be minimized.

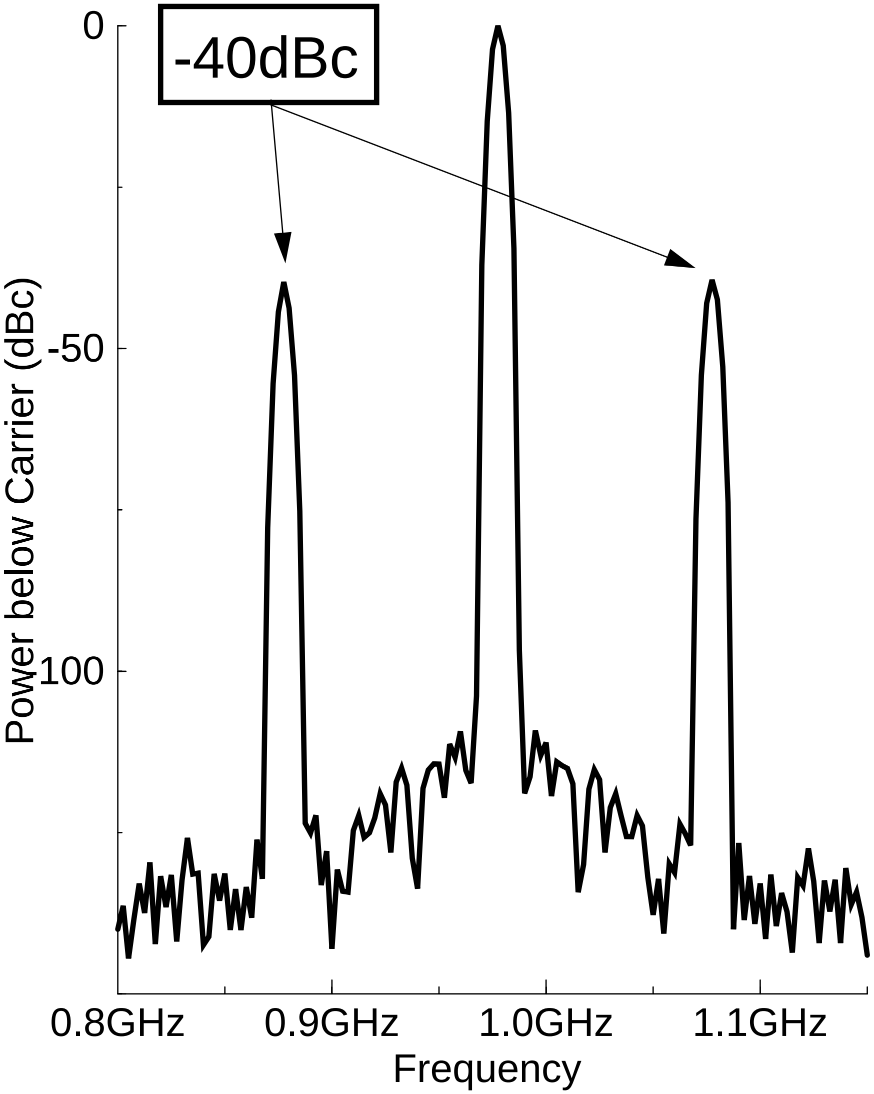
Ring Oscillator with an Asymmetric Stage



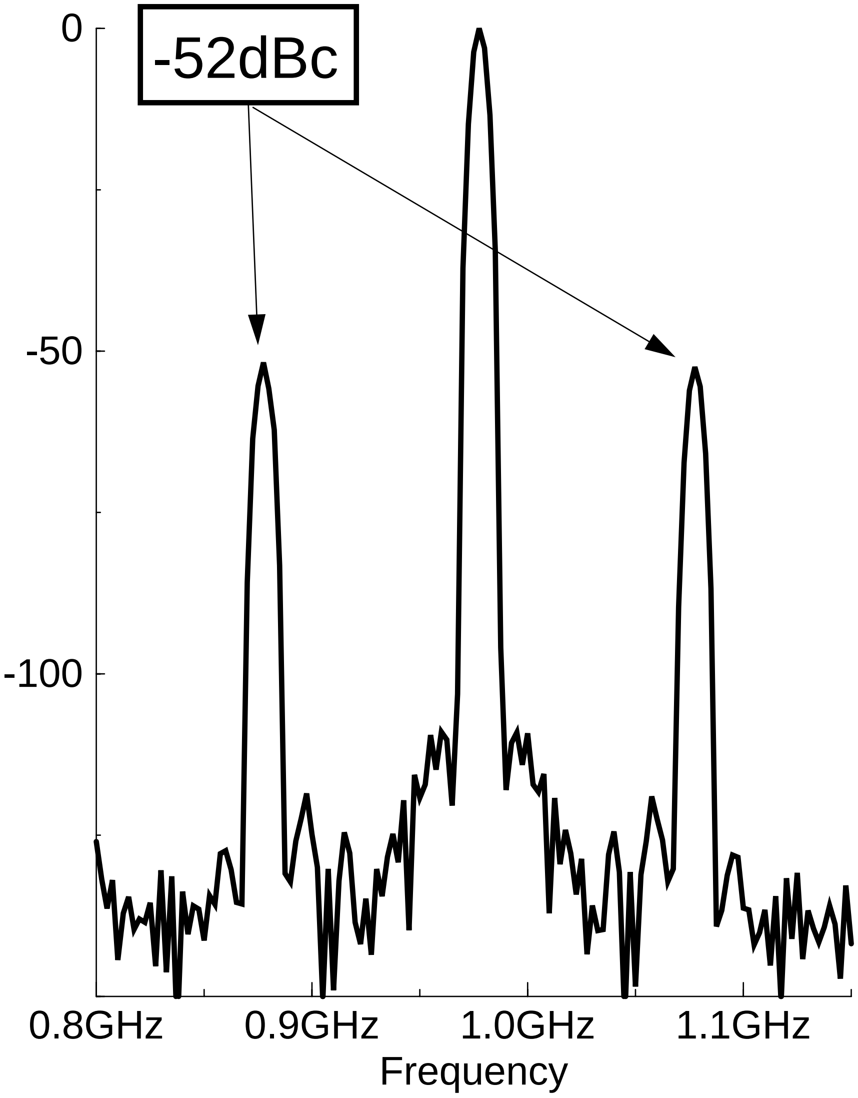
The effect of asymmetry can be seen by comparing the effect of low frequency injection into symmetric and asymmetric nodes.

Low Frequency Upconversion

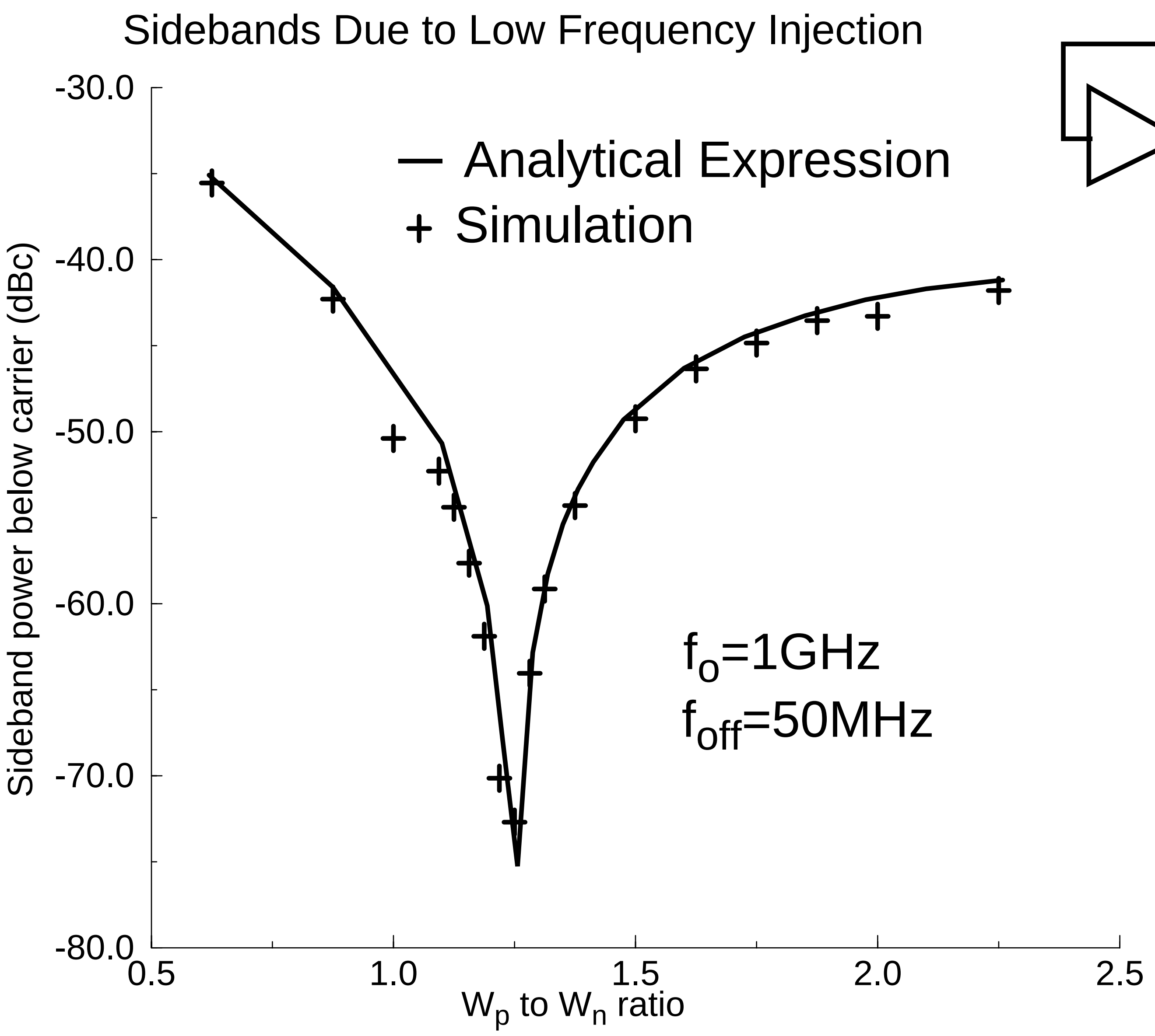
Injection into Asymmetric Node

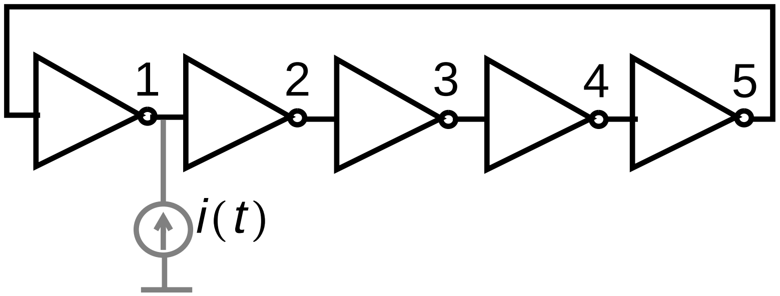


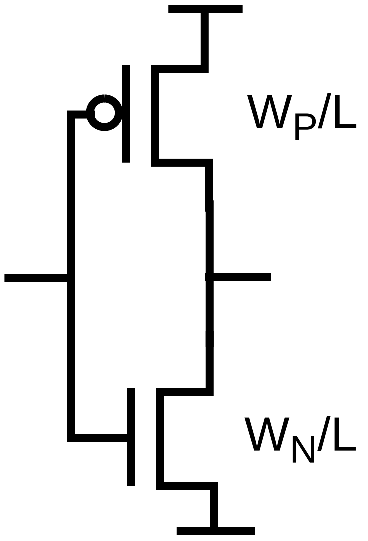
Injection into Symmetric Node



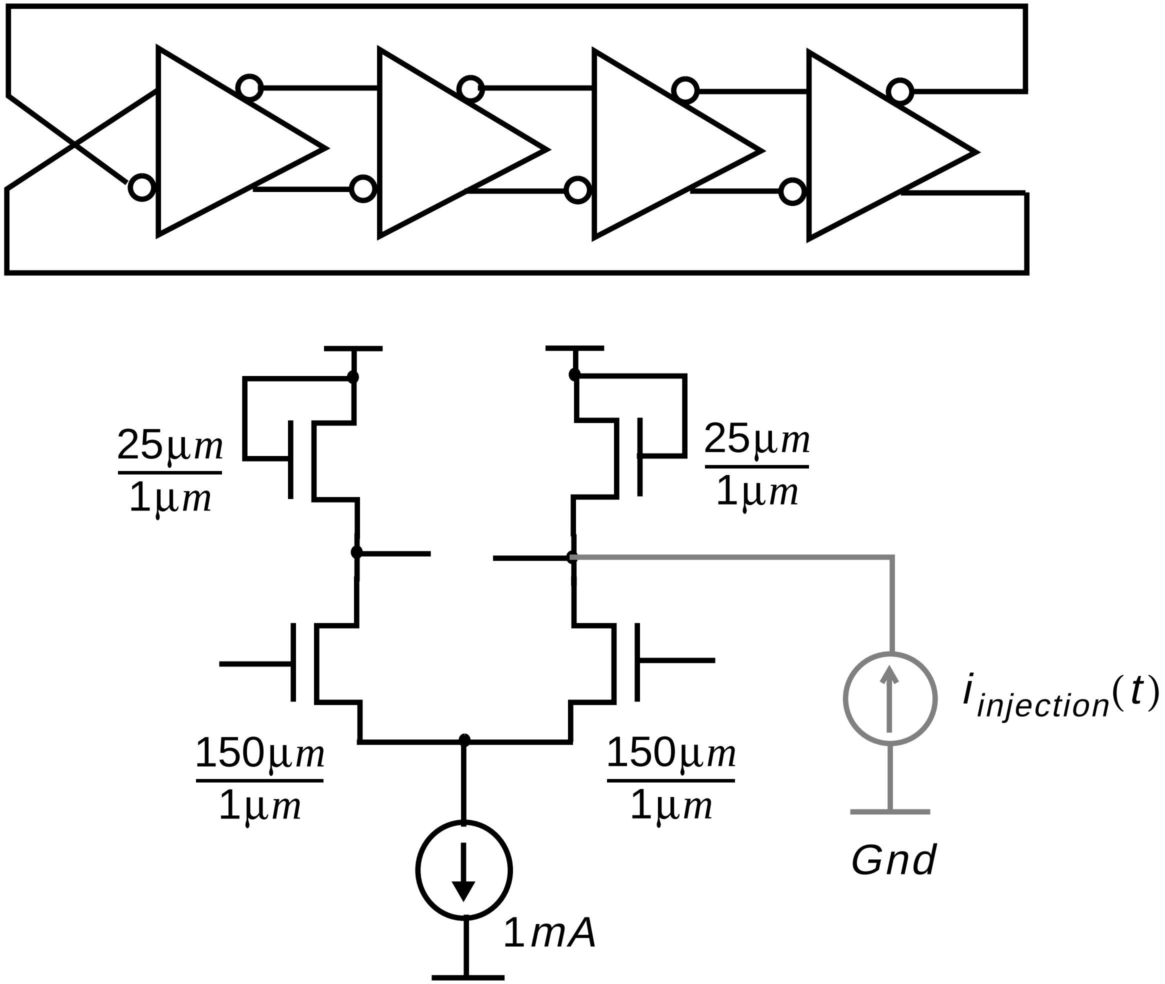
Effect of Rise and Fall Time Symmetry







Effect of Differential Symmetry

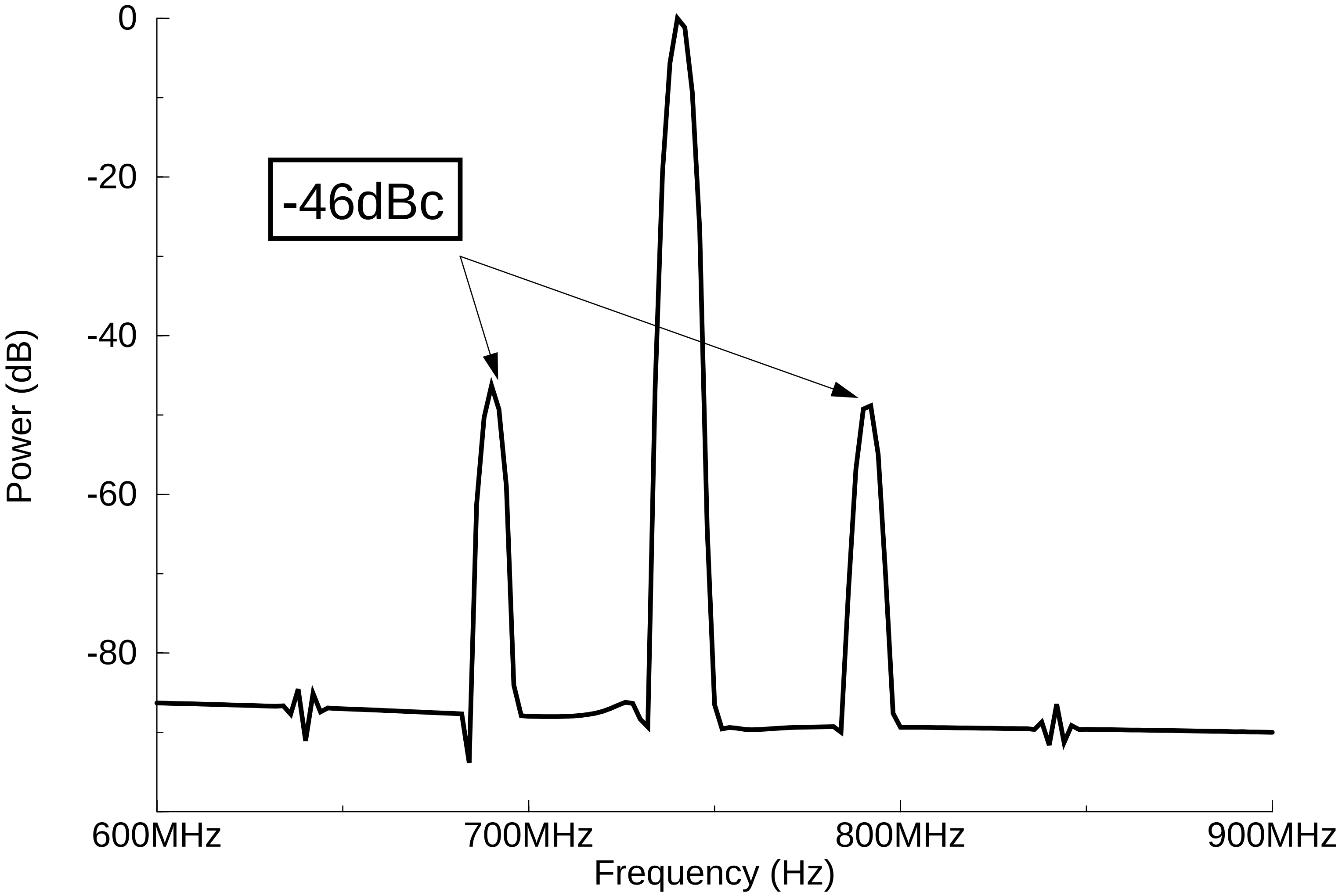


The noise sources on each of the differential nodes are not fully correlated.

It is the symmetry of the half circuit that matters.

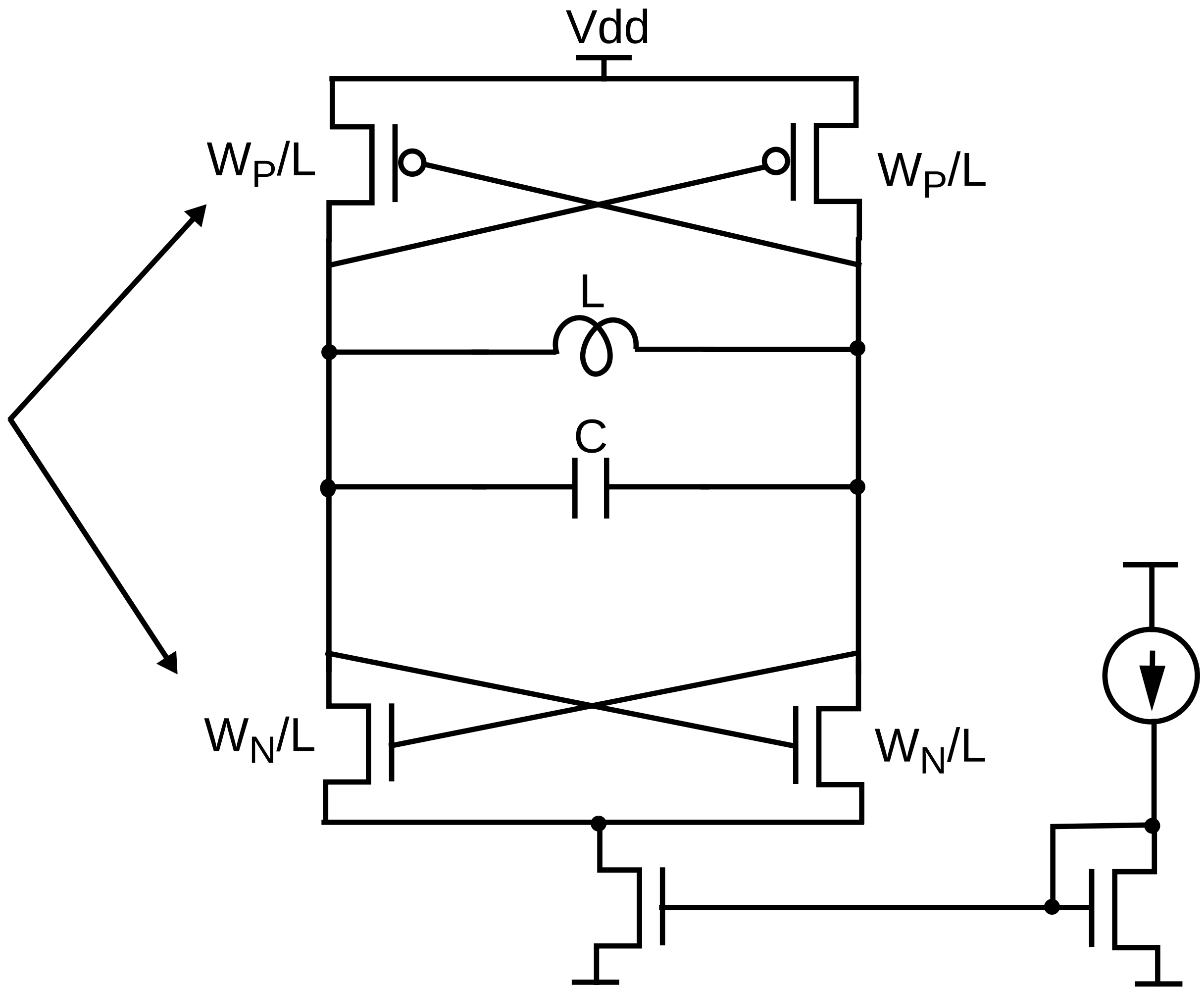
Effect of Differential Symmetry

Low Frequency Current Injection into Differential Ring



Differential symmetry does not automatically eliminate the low frequency upconversion.

A Symmetric LC Oscillator



Adjust ratios for symmetry

Possible to Adjust Symmetry Properties of the Waveform

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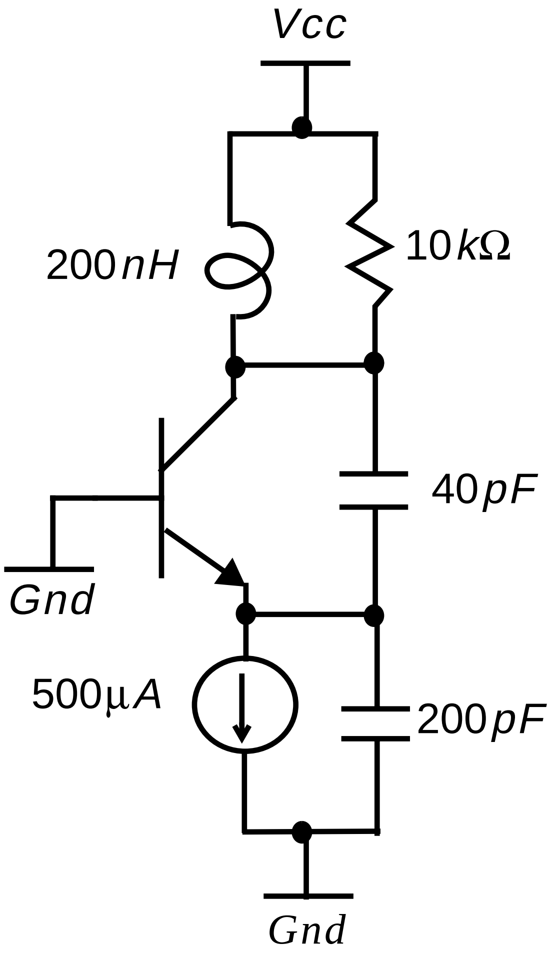
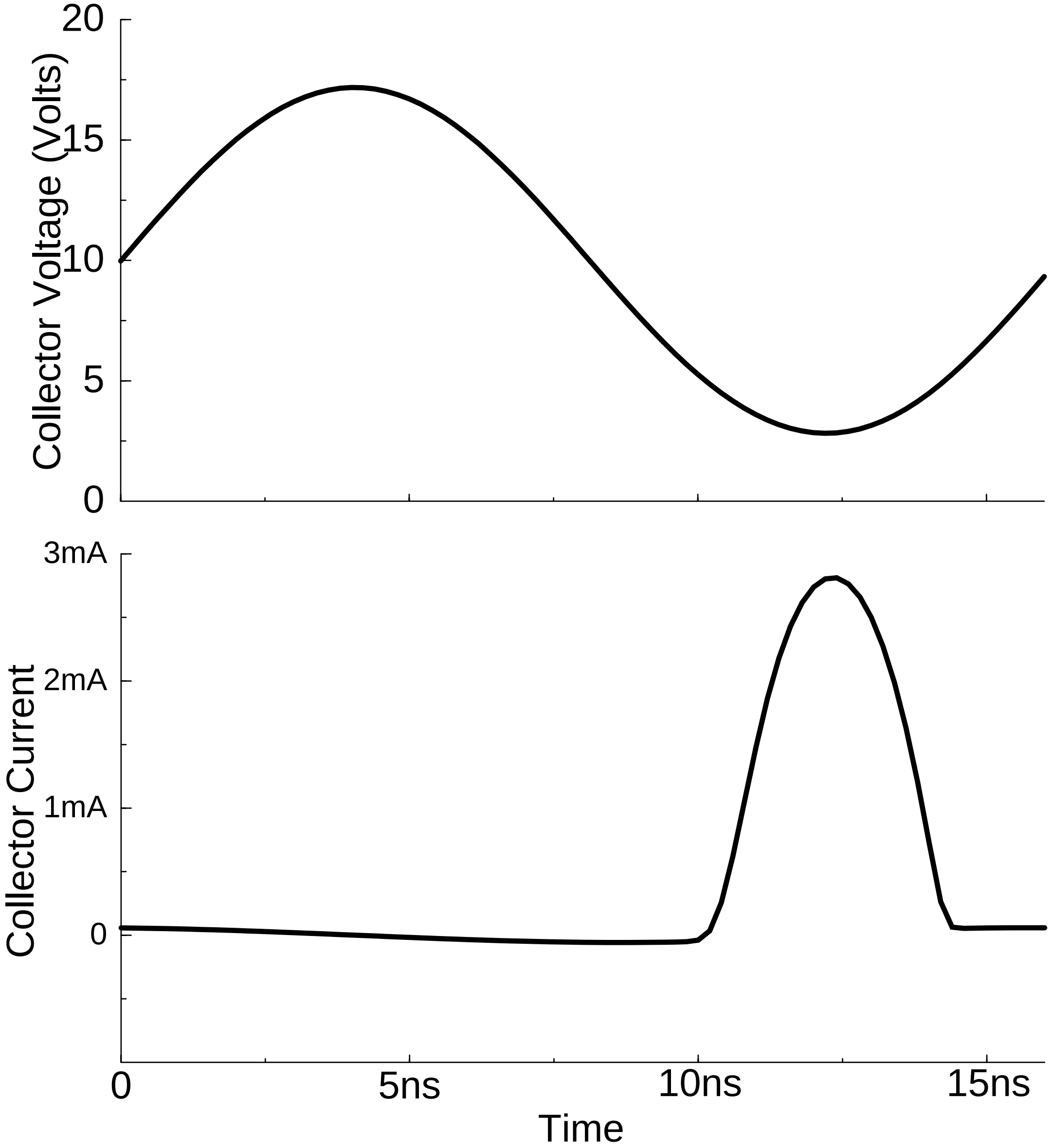
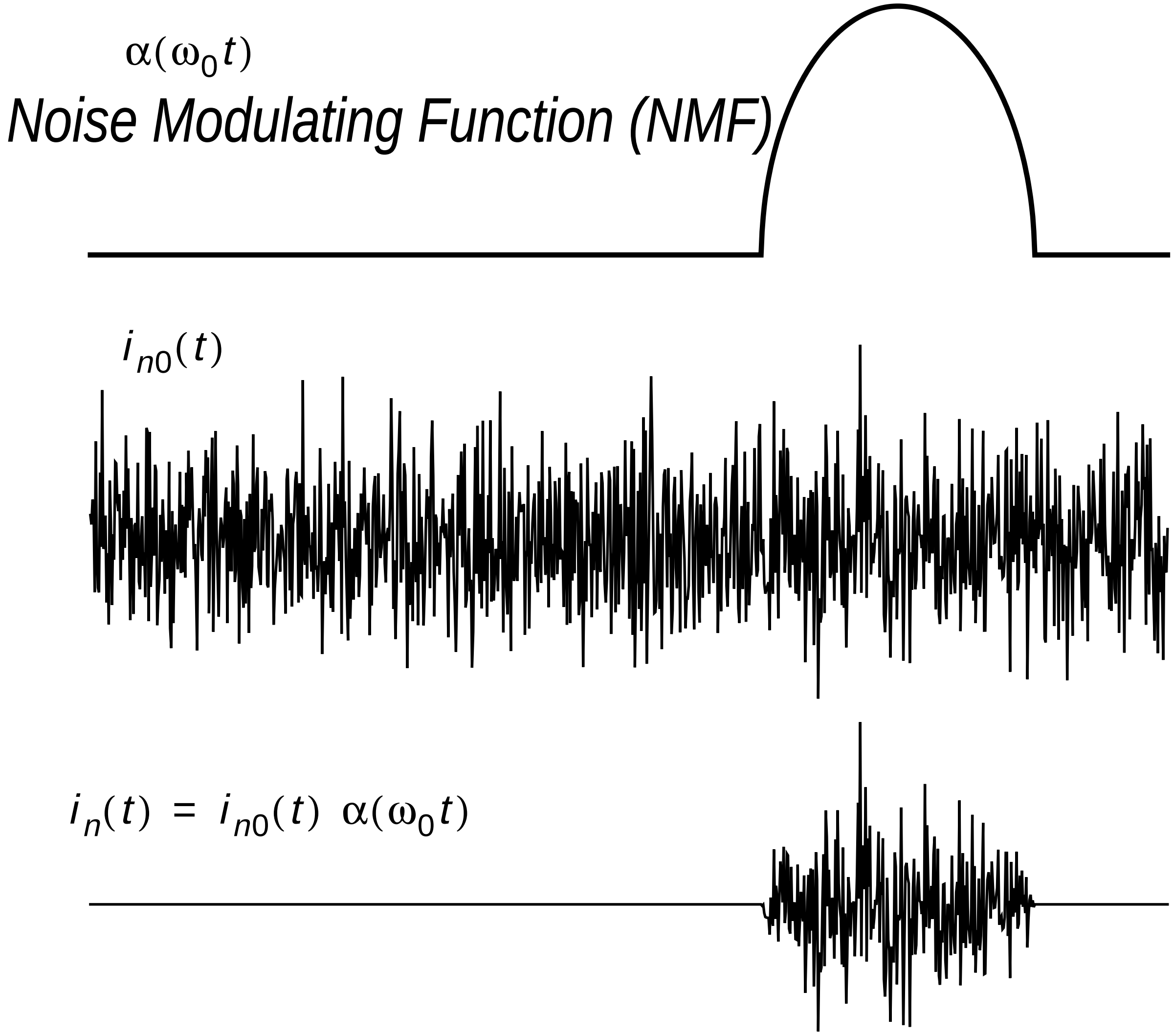
Time Varying Current in Colpitts Oscillator





Cyclostationary Properties, Time Domain



$$\phi(t) = \int_{-\infty}^{t} i_n(\tau)\frac{\Gamma(\omega_0\tau)}{q_{max}}d\tau$$

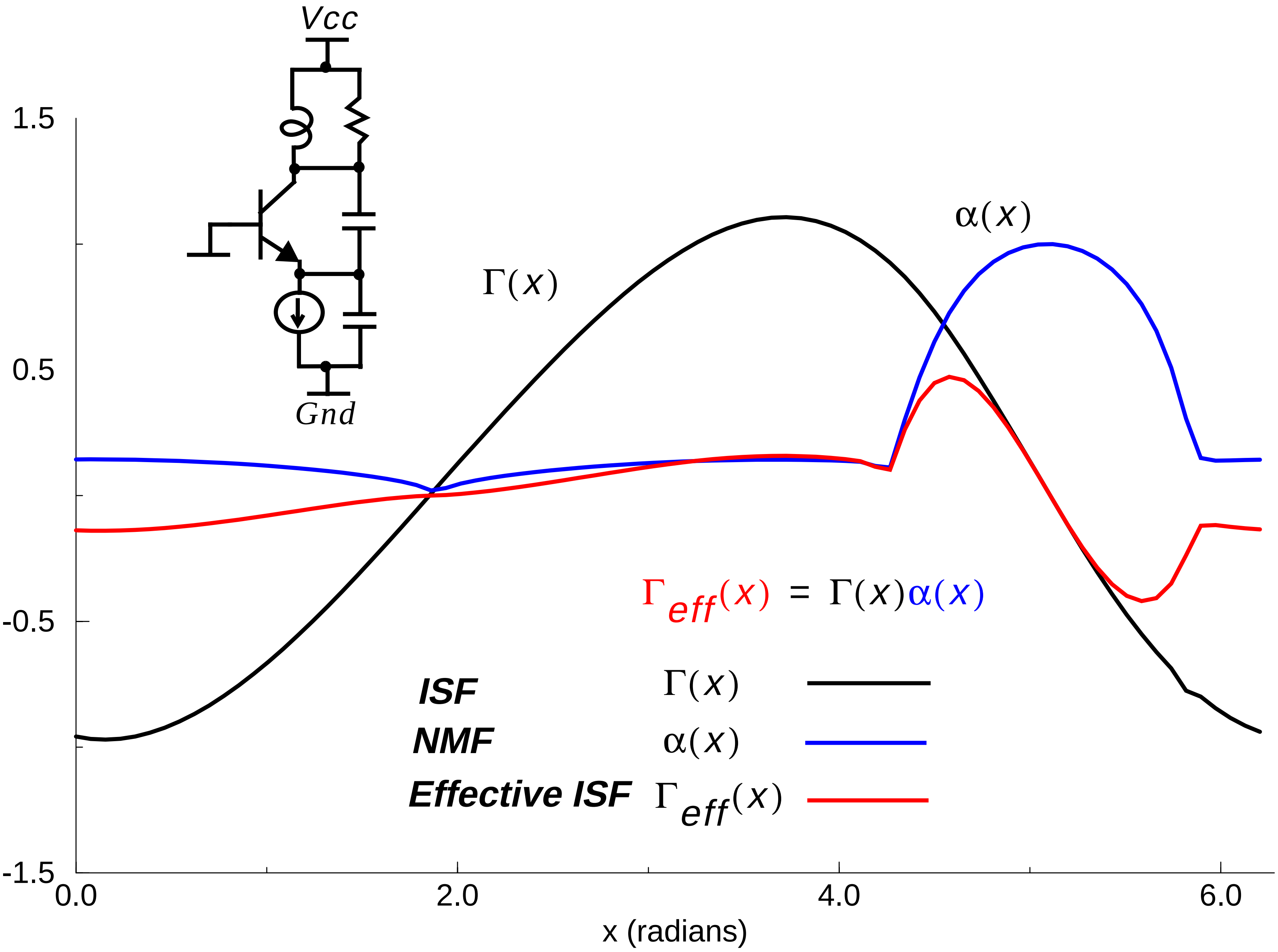
$$= \int_{-\infty}^{t} i_{n0}(\tau)\frac{\alpha(\omega_0\tau)\Gamma(\omega_0\tau)}{q_{max}}d\tau$$

Effective ISF:

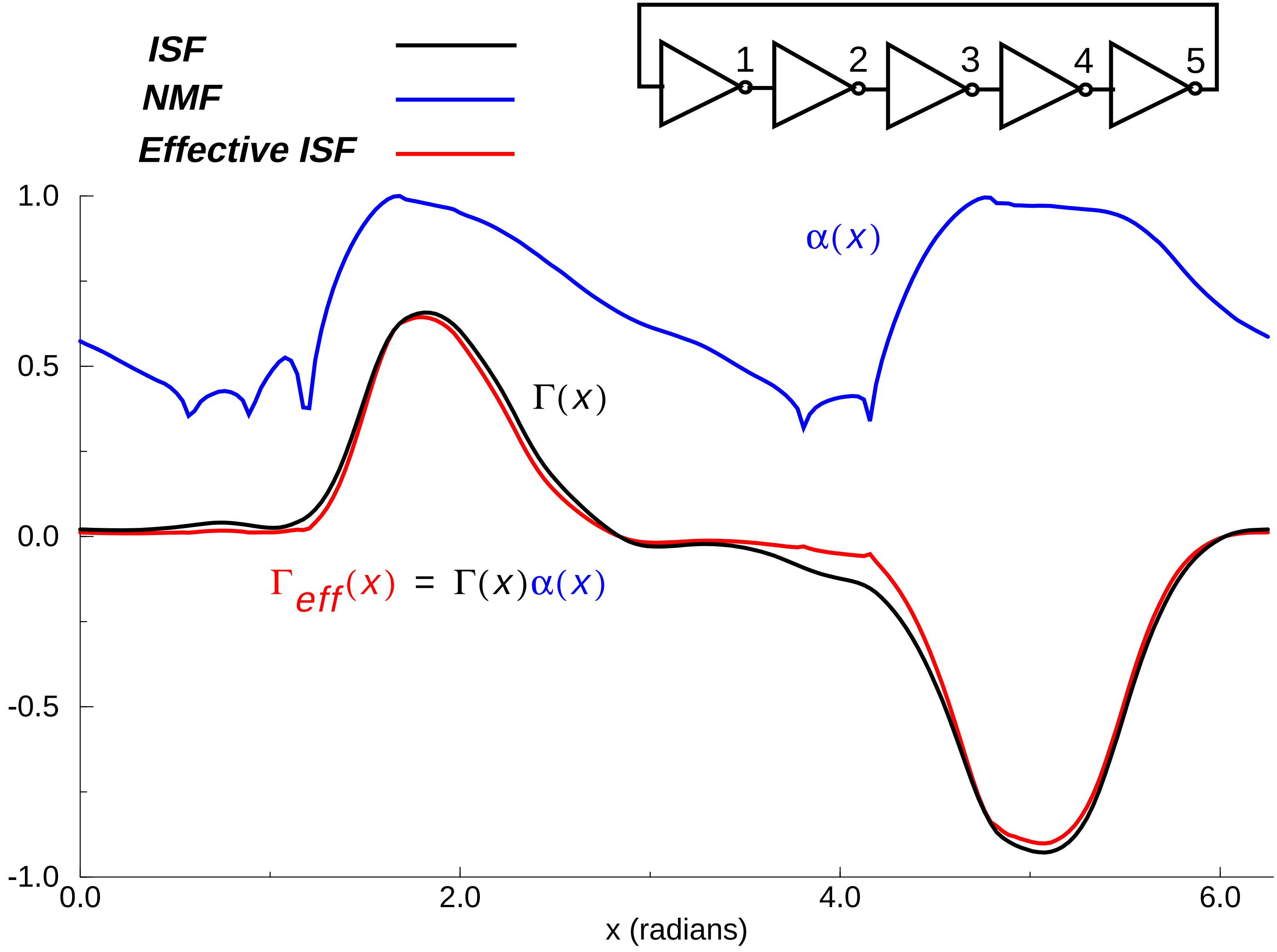
$$\Gamma_{eff}(x) = \Gamma(x)\cdot\alpha(x)$$

A cyclostationary source can be modeled as stationary with a new ISF.

Colpitts Oscillator



5 Stage Ring Oscillator



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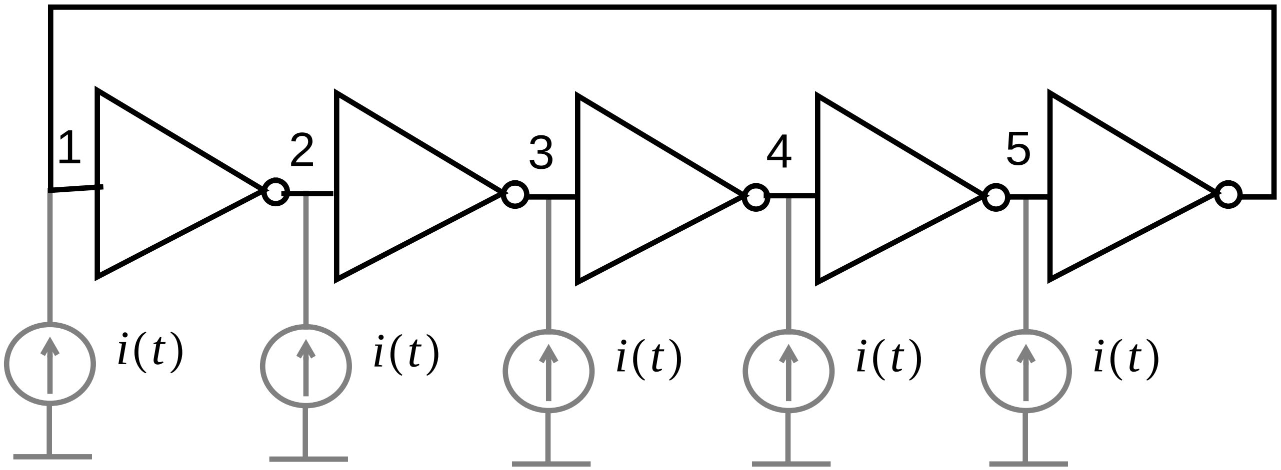
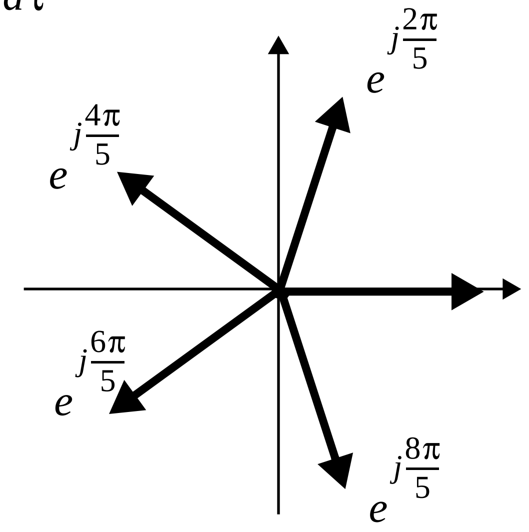
Fully Correlated Sources



Superposition: $\phi_{total}(t) = \sum_{k=1}^{N} \phi(t) = \int_{-\infty}^{t} \frac{i(\tau)}{q_{max}}\left[\sum_{k=1}^{N} \Gamma_k(\omega_0 \tau)\right] d\tau \approx \frac{Nc_0}{q_{max}} \int_{-\infty}^{t} i(\tau) d\tau$

Similar stages: $\Gamma_k(\omega_o t) = \Gamma\left(\omega_o t + \frac{2\pi k}{N}\right)$

$$\sum_{k=1}^{N} \Gamma_k(\omega_0 \tau) \approx N c_0$$



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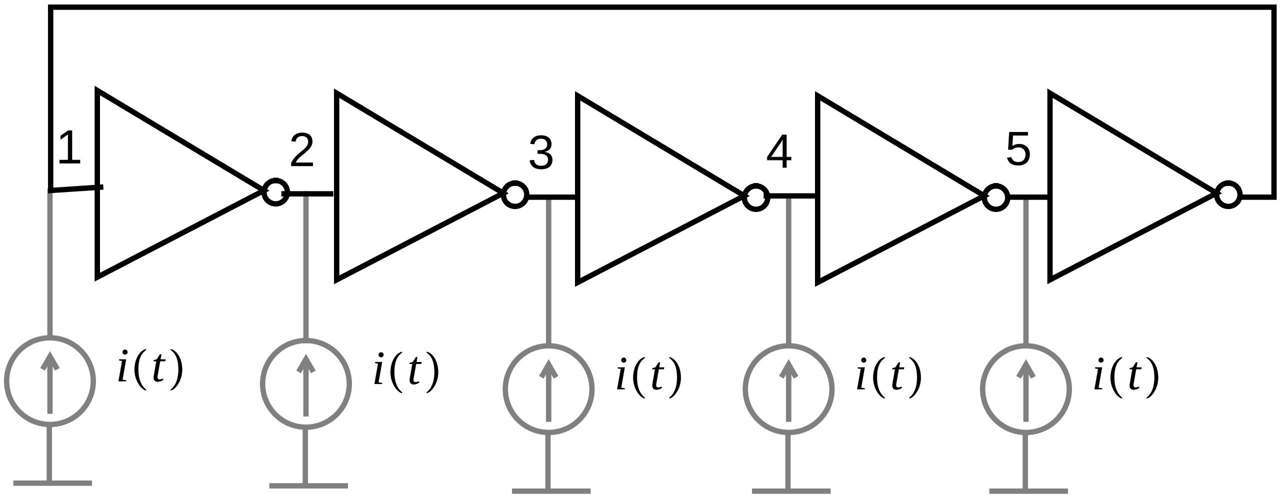

$$\phi_{total}(t) \approx \frac{Nc_0}{q_{max}} \int_{-\infty}^{t} i(\tau)d\tau$$

Only the low frequency portion of substrate and supply noise is important, provided:

1. Stages and loadings are the same
2. The noise sources are identical

This is good news since c_0 can be significantly reduced by adjusting the symmetry.

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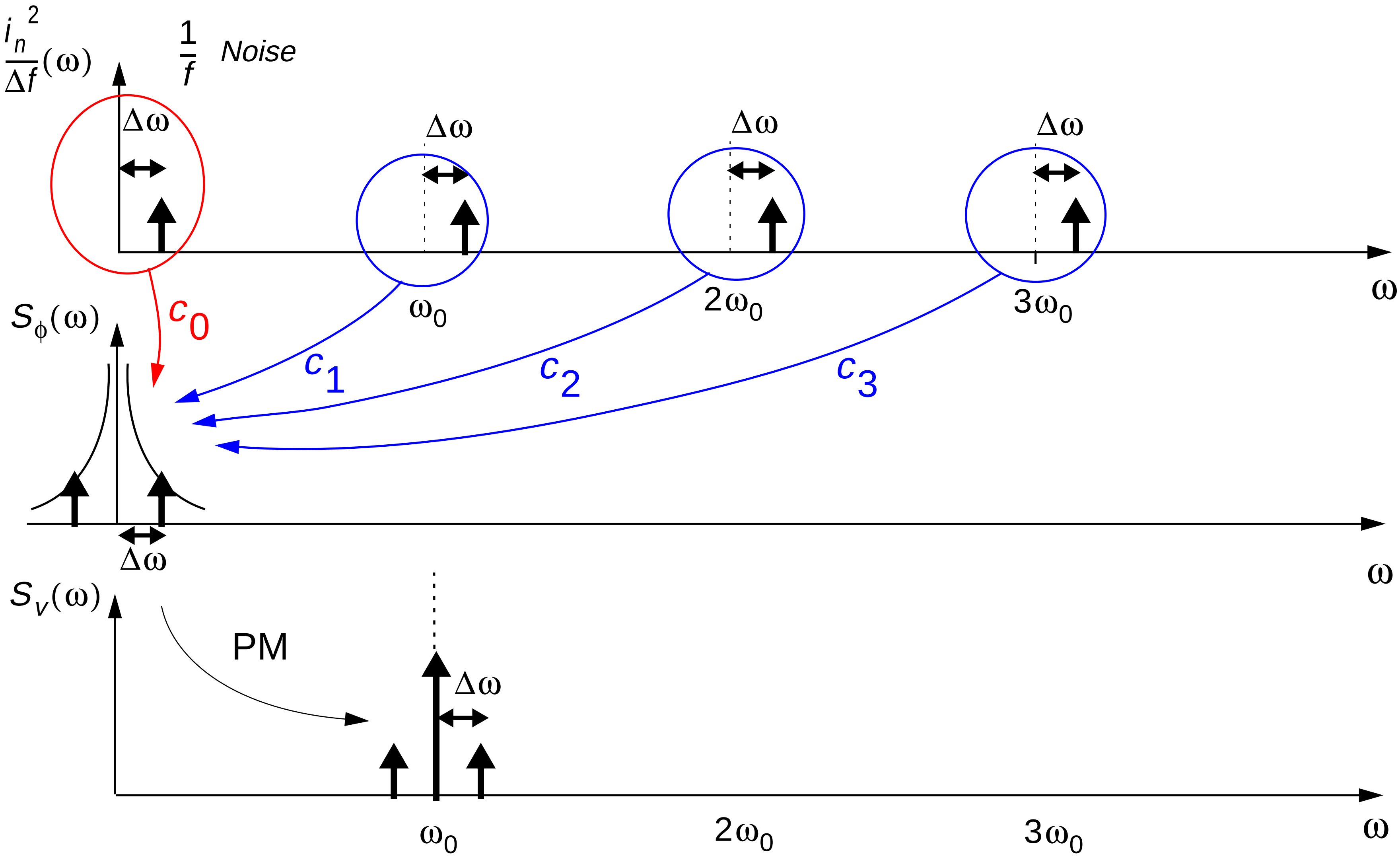
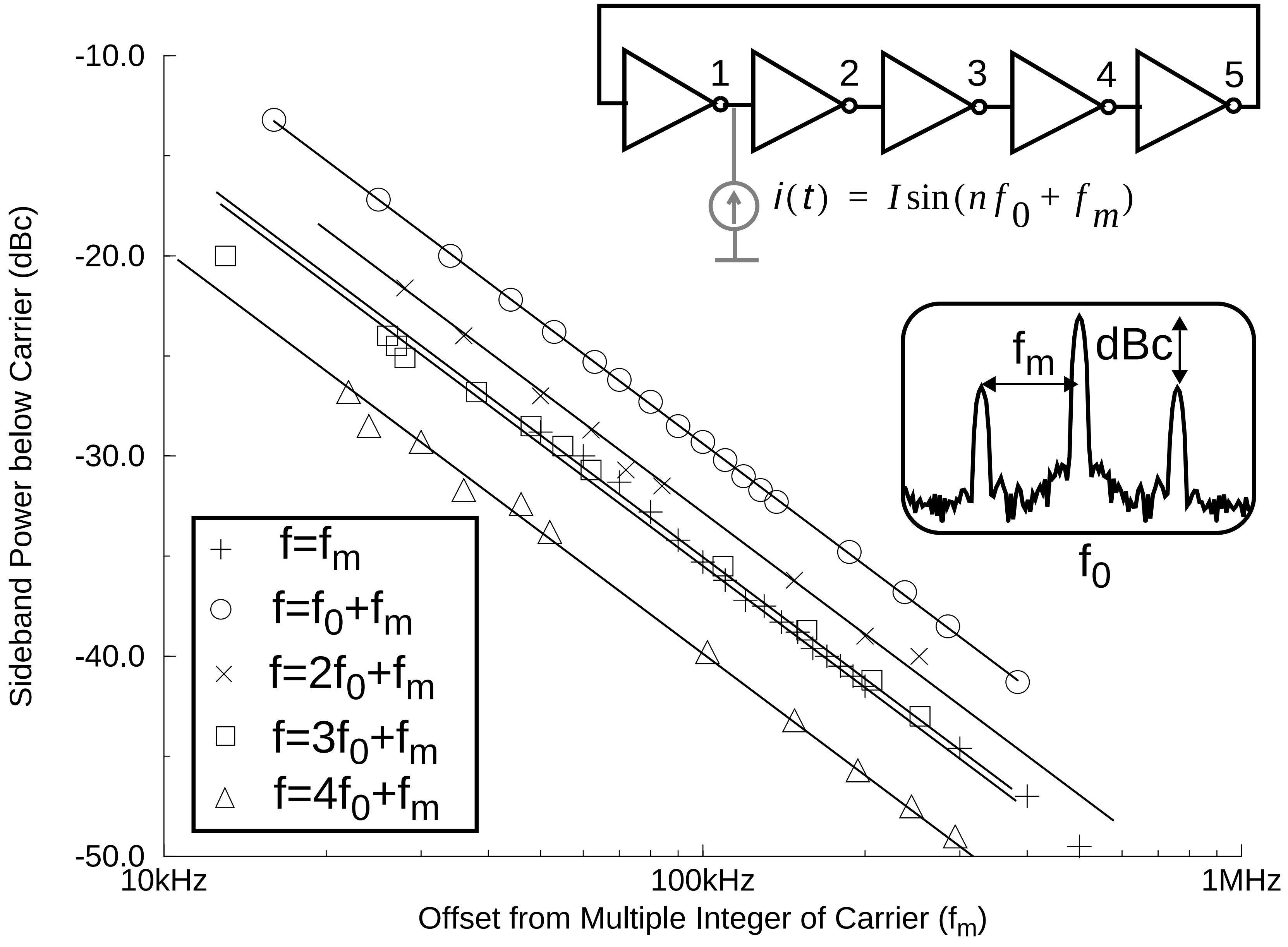
Noise Contributions from $n\omega_o$

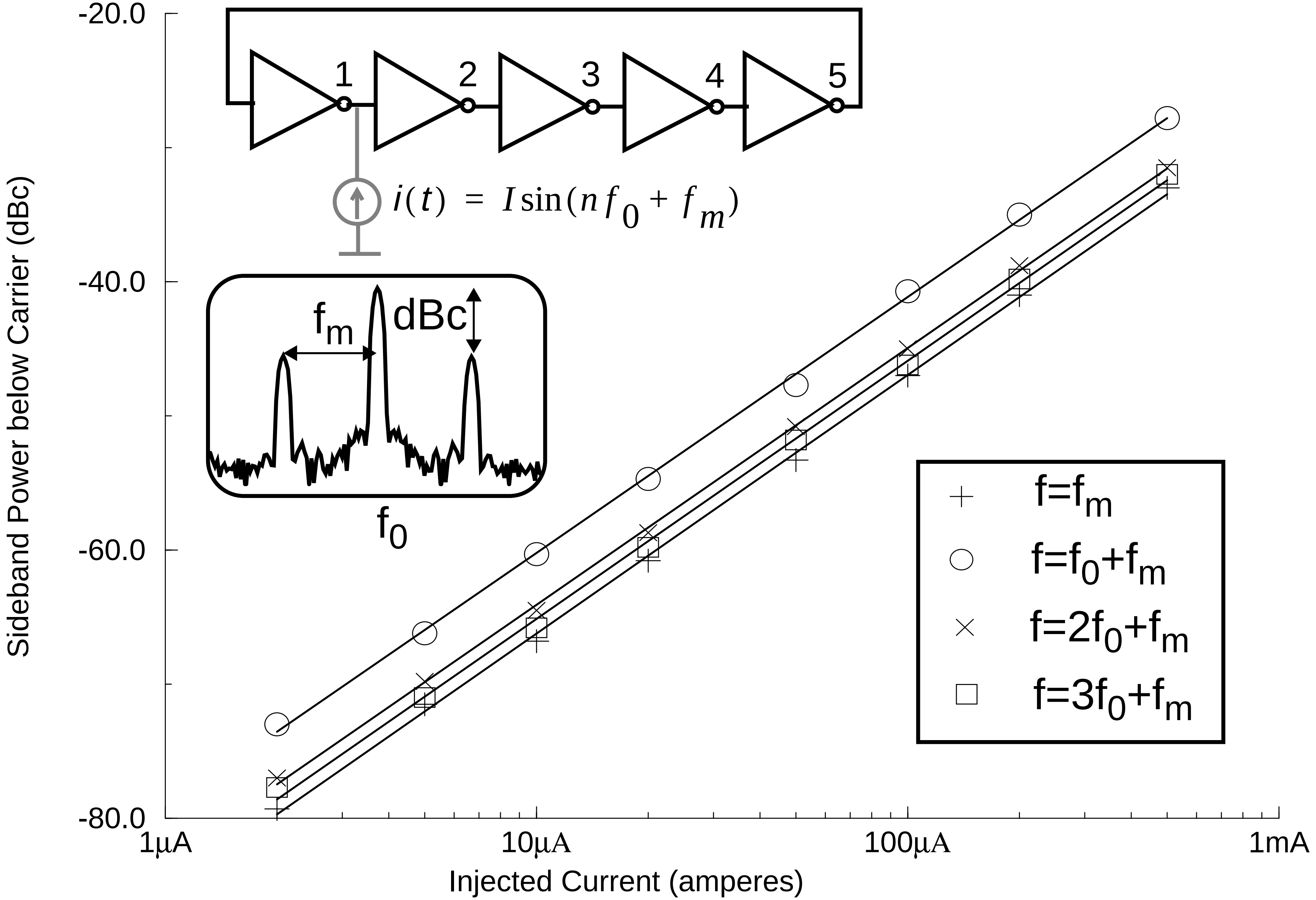
$$\phi(t) = \frac{1}{q_{max}}\left[c_0\int_{-\infty}^{t} i(\tau)d\tau + \sum_{n=1}^{\infty} c_n\int_{-\infty}^{t} i(\tau)\cos(n\omega\tau)d\tau\right]$$



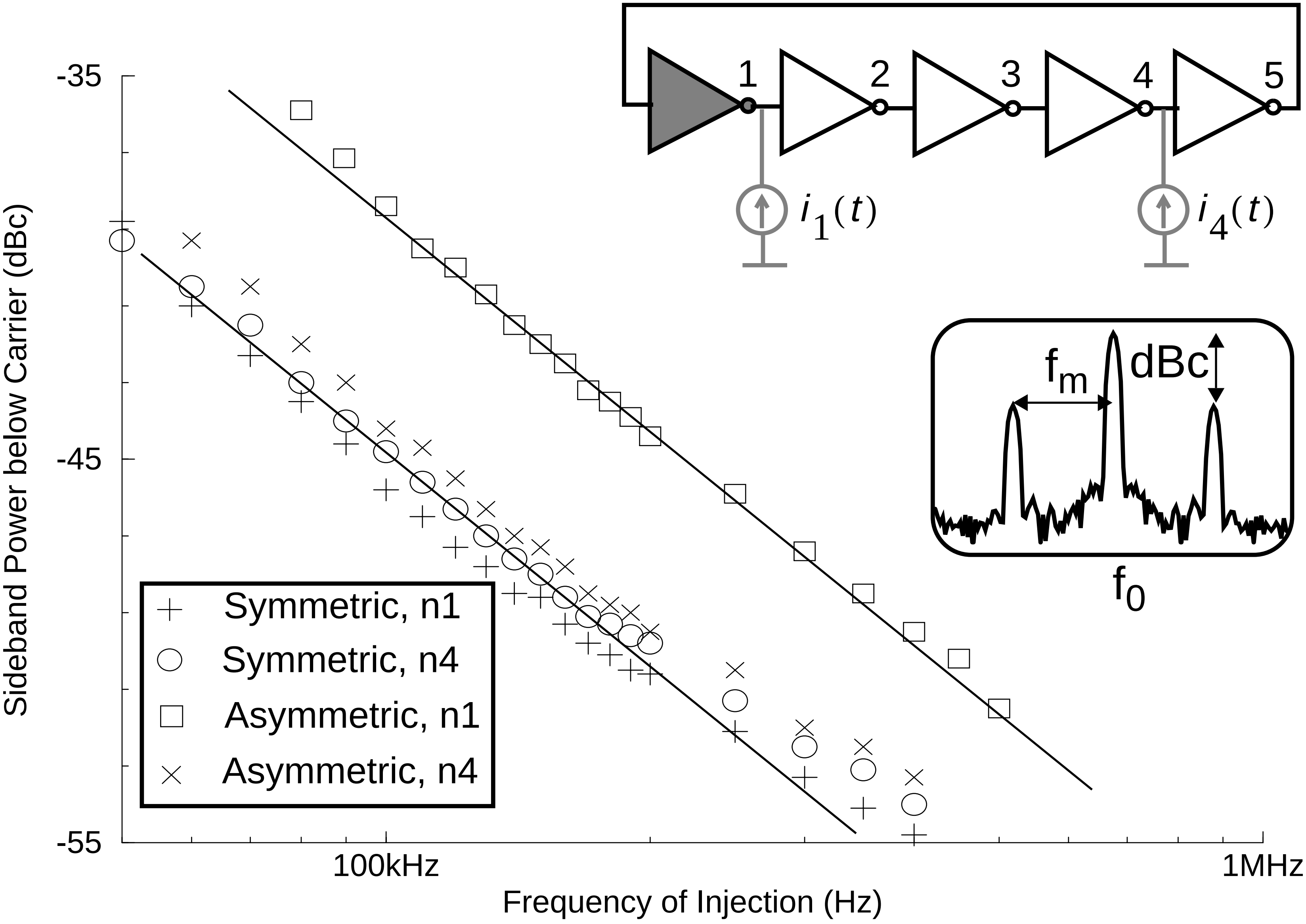
Injection at Integer Multiples of f_0



Sideband Power vs. Injection Current

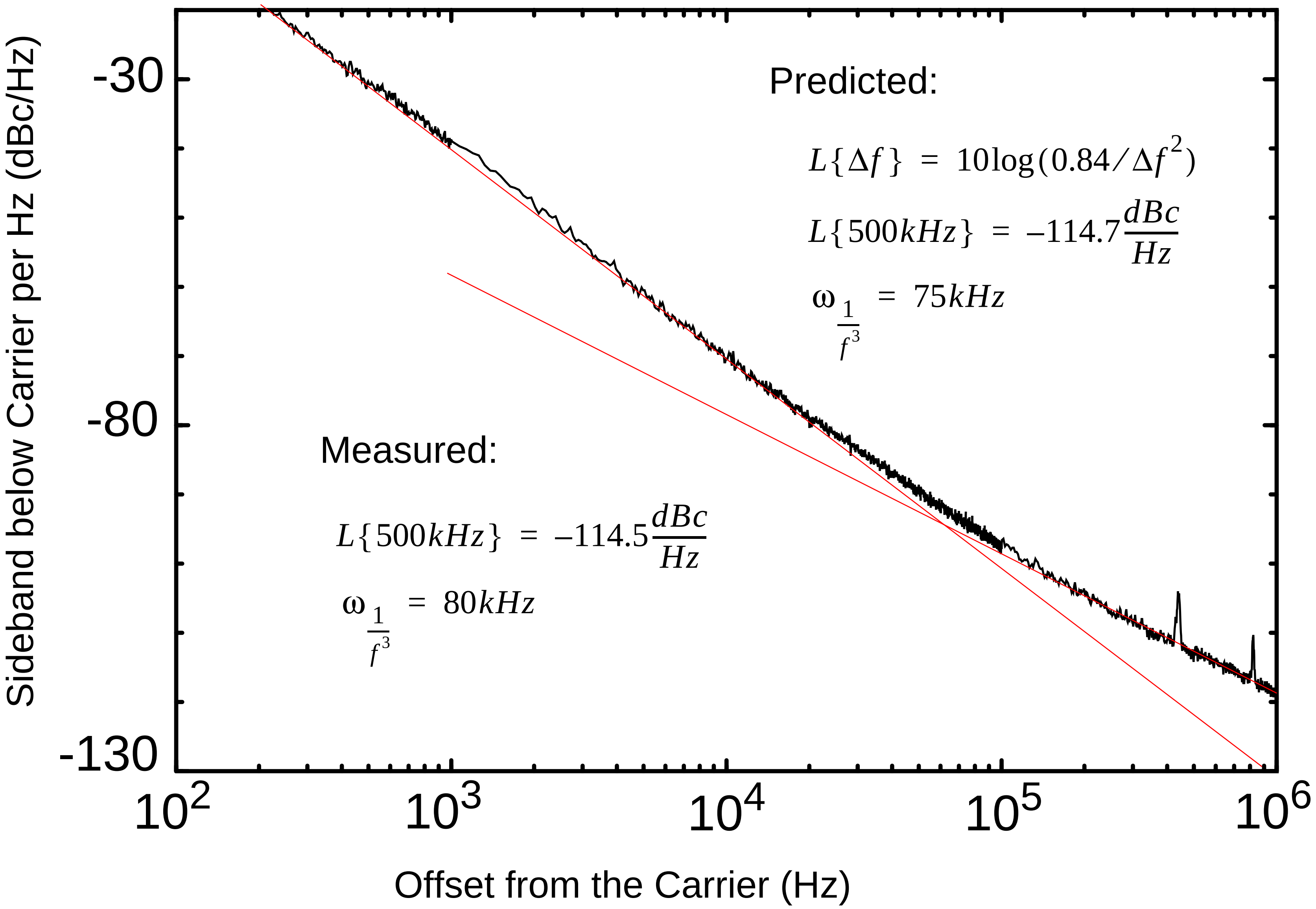


Symmetric vs. Asymmetric Ring Oscillator



5-Stage Single-Ended Ring Oscillator

f_0=232MHz, 2μm Technology



Predicted:

$$L\{\Delta f\} = 10\log(0.84/\Delta f^2)$$

$$L\{500kHz\} = -114.7\frac{dBc}{Hz}$$

$$\omega_{\frac{1}{f^3}} = 75kHz$$

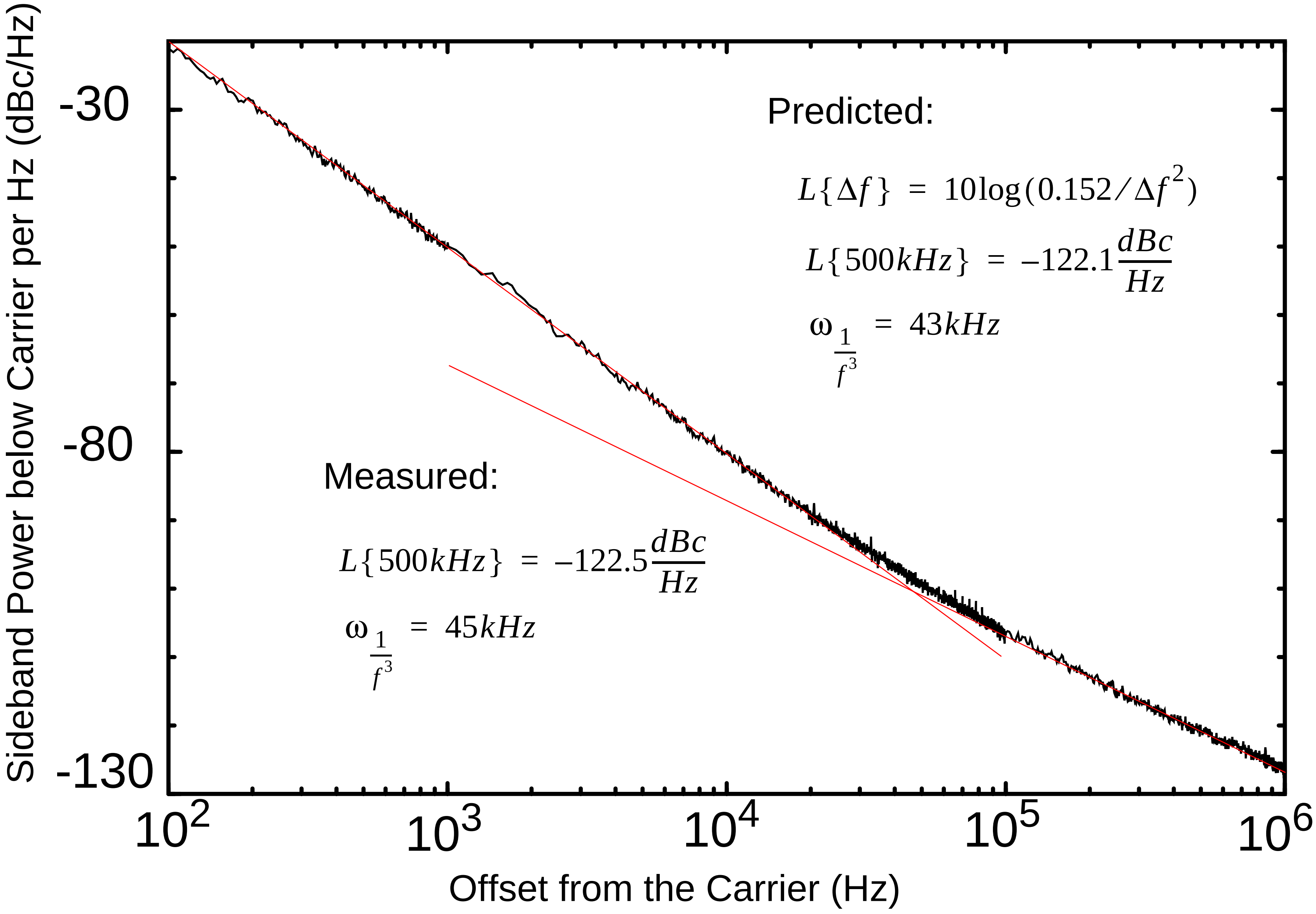
Measured:

$$L\{500kHz\} = -114.5\frac{dBc}{Hz}$$

$$\omega_{\frac{1}{f^3}} = 80kHz$$

11-Stage Single-Ended Ring Oscillator

f_0=115MHz, 2µm Process



Predicted:

$$L\{\Delta f\} = 10\log(0.152/\Delta f^2)$$

$$L\{500kHz\} = -122.1\frac{dBc}{Hz}$$

$$\omega_{\frac{1}{f^3}} = 43kHz$$

Measured:

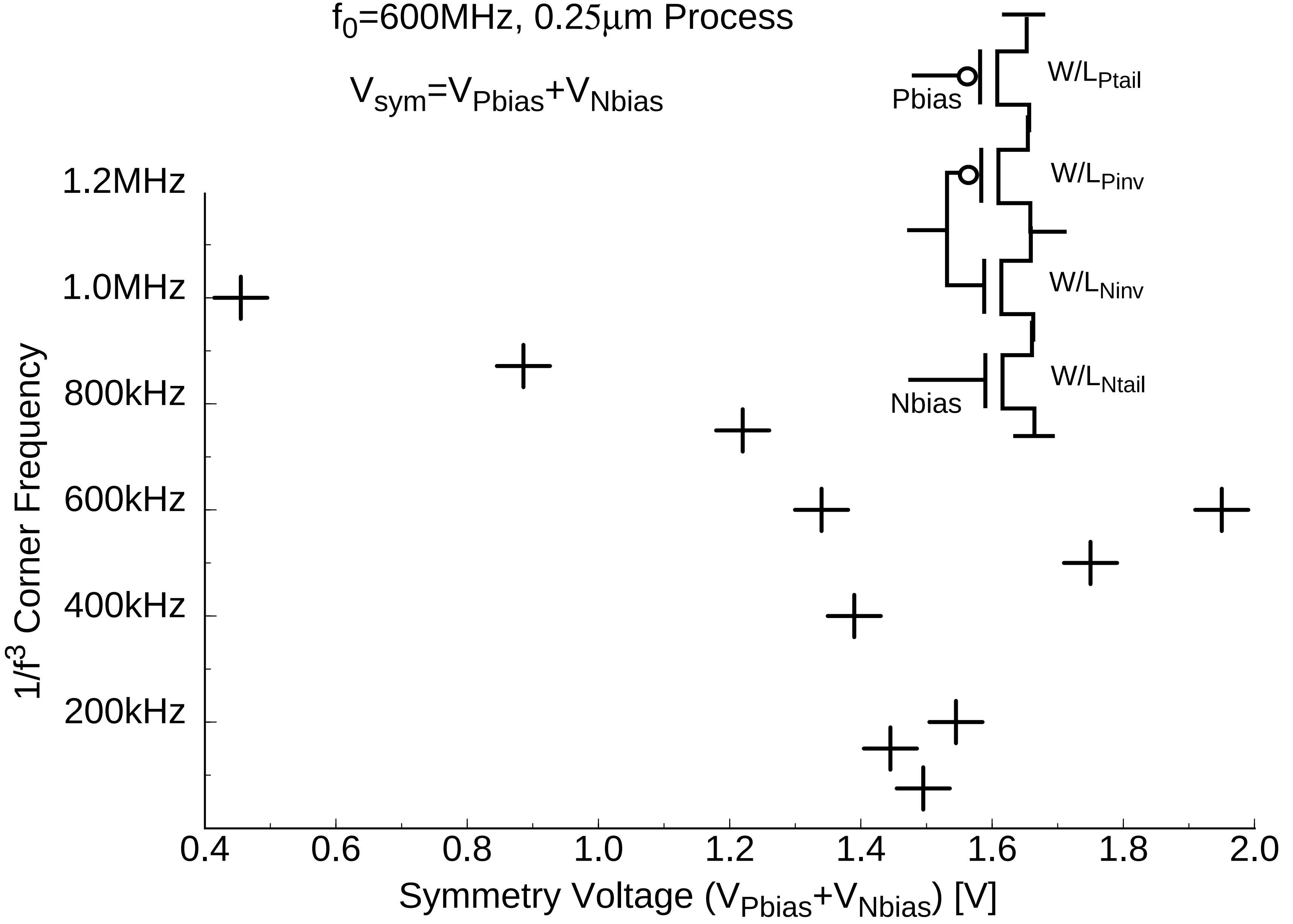
$$L\{500kHz\} = -122.5\frac{dBc}{Hz}$$

$$\omega_{\frac{1}{f^3}} = 45kHz$$

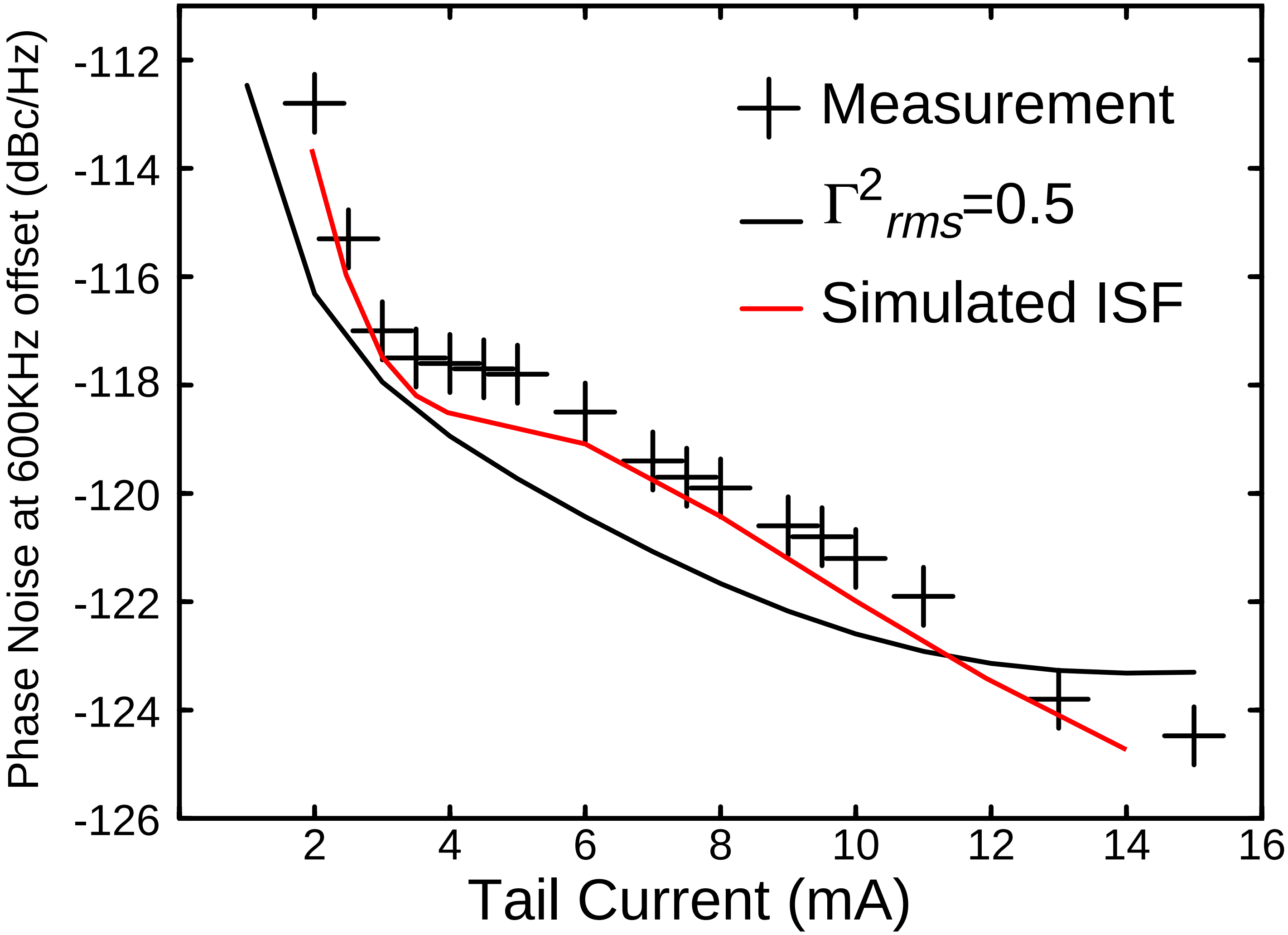
9-Stage Current Starved Single-Ended VCO

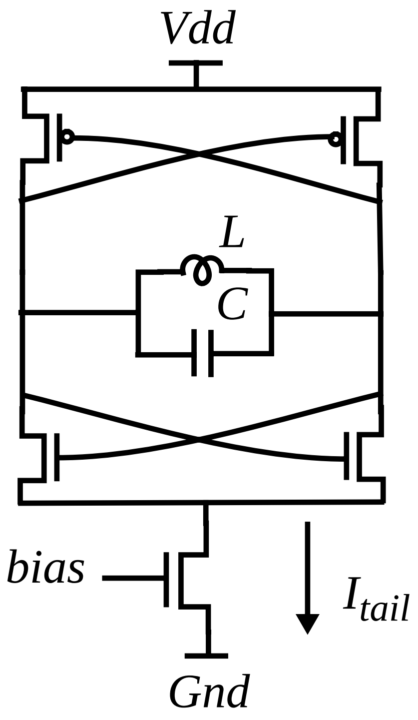
f_0=600MHz, 0.25μm Process

$V_{sym}=V_{Pbias}+V_{Nbias}$



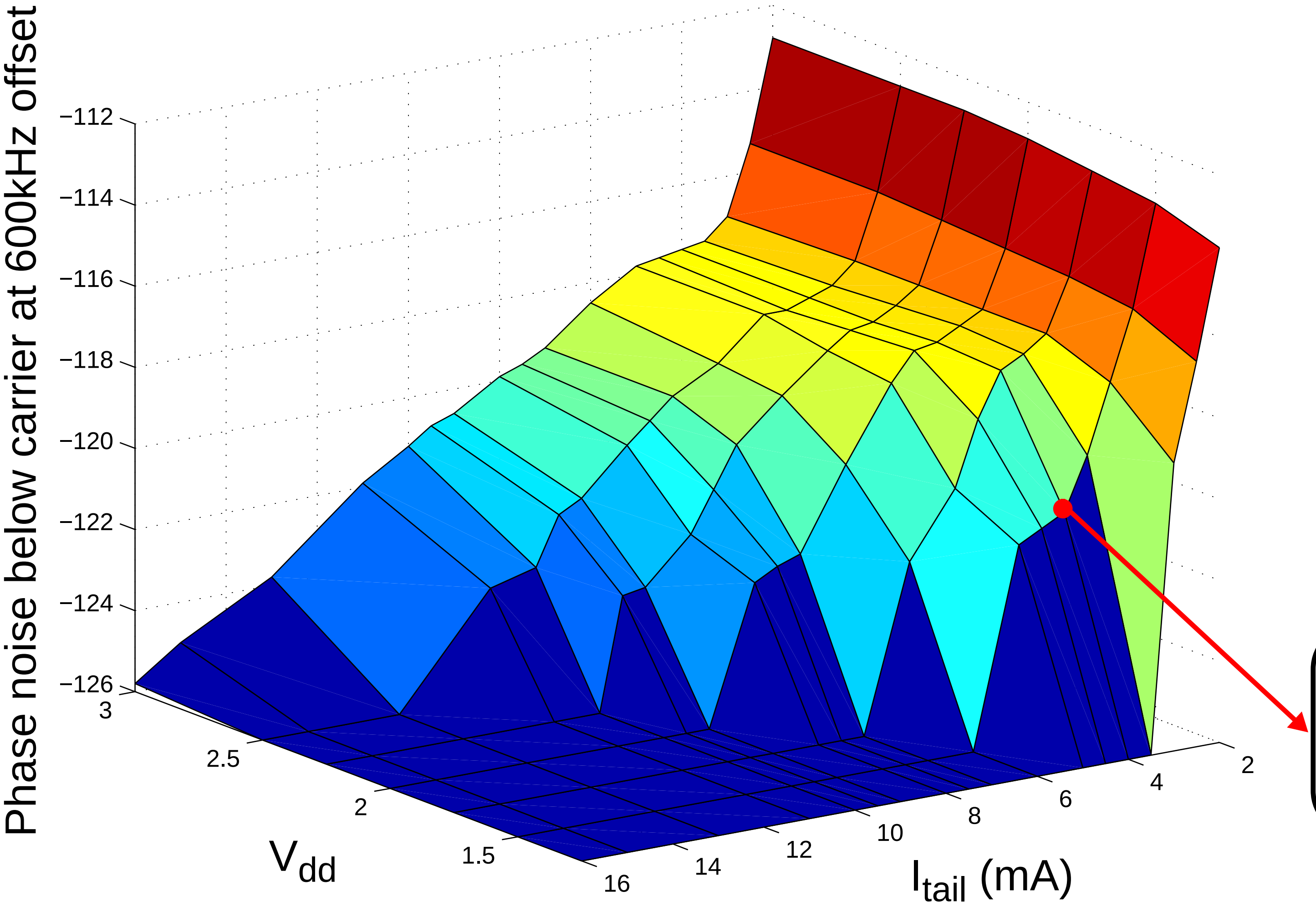
Complementary Cross-Coupled LC Oscillator

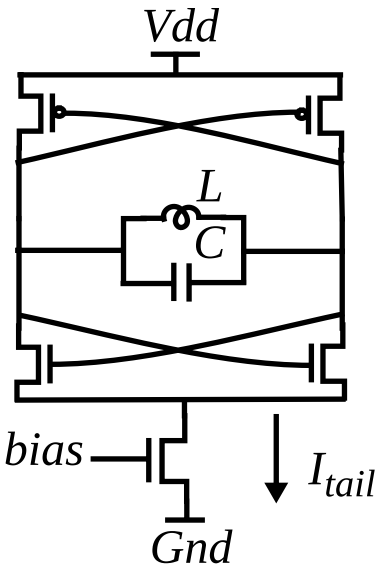

f_0=1.8GHz, 0.25μm Process (V_{DD} =3V)



Complementary Cross-Coupled VCO

f_0=1.8GHz, 0.25µm Process

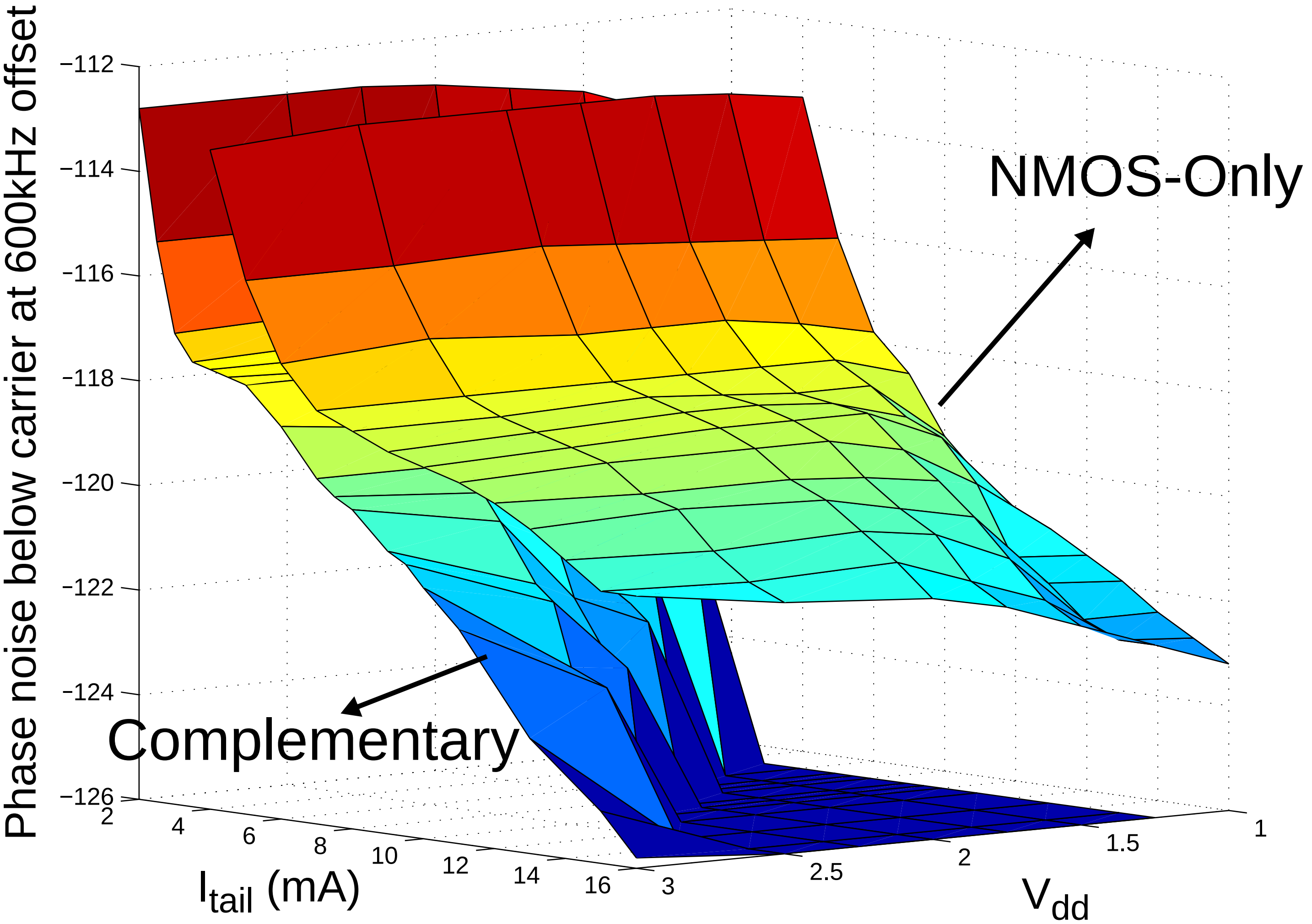


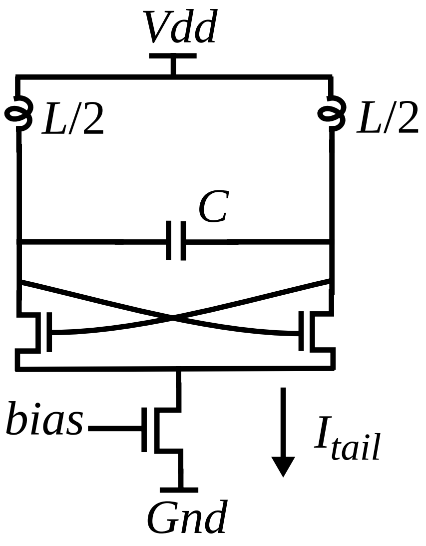


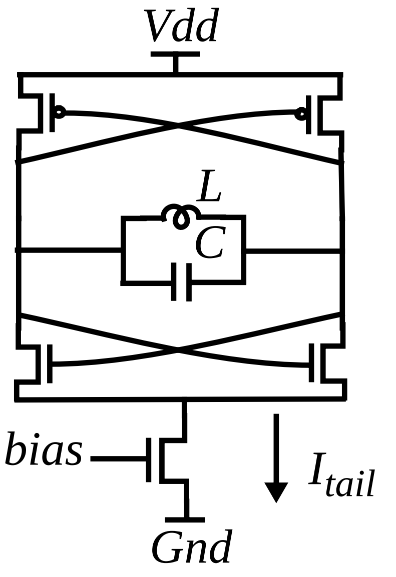
f_0=1.8GHz
P=6mW
-121dBc/Hz@600kHz

Complementary vs. NMOS-Only VCO

f_0=1.8GHz, 0.25μm Process

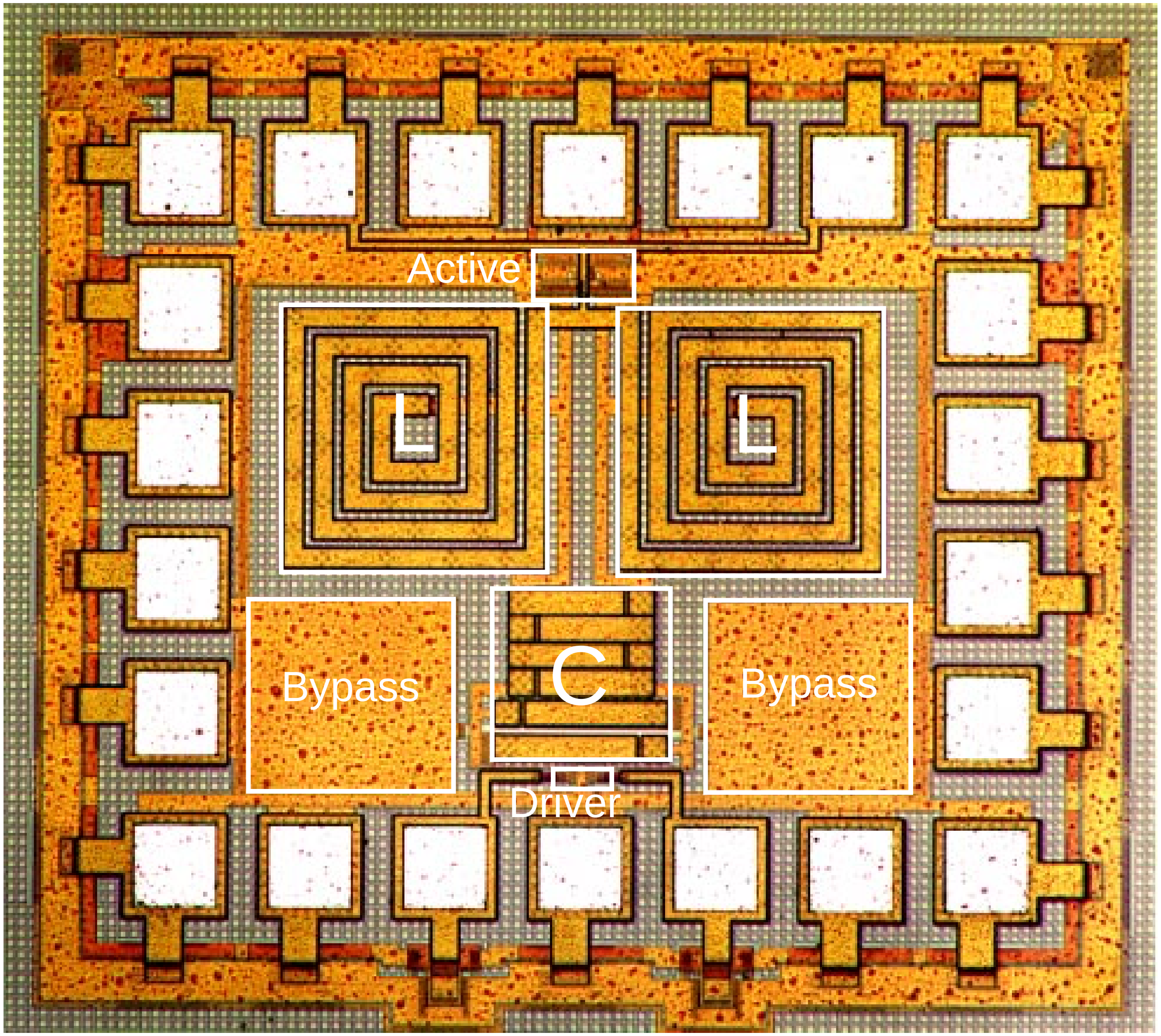






Email: hajimiri@smirc.stanford.edu

Die Photo of the Complementary Oscillator



0.25µm Process

700µm x 800µm

Pad limited

Conclusion and Contributions

A new general model for phase noise is introduced, which:

- is independent of the topology of the oscillator,
- is useful both as an analysis and a design tool,
- is valid for arbitrary sources of noise and interference,
- predicts the effect of symmetry on the upconversion of 1/*f* noise,
- incorporates cyclostationary noise sources naturally,
- predicts the effect of correlation on phase noise,
- reduces to previously existing models as special cases,
- shows agreement among theory, simulation and measurements.

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DEDICATED TO ALL MY TEACHERS.