Phase Noise in Multi-Gigahertz CMOS Ring Oscillators

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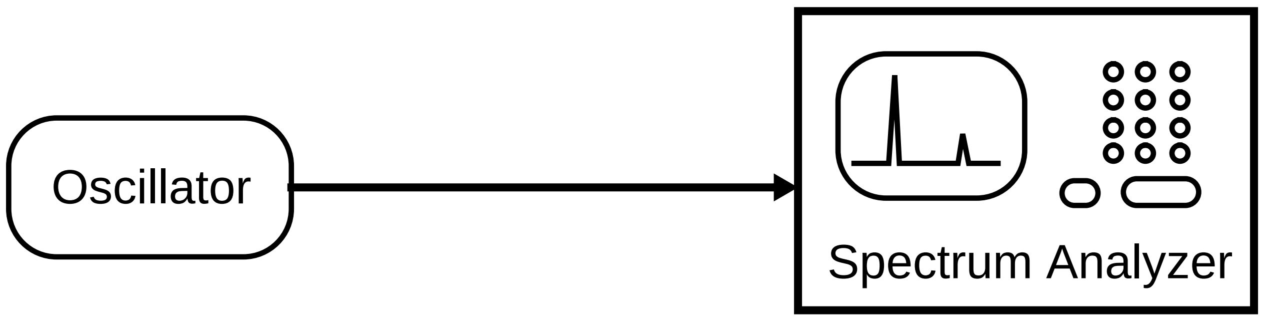
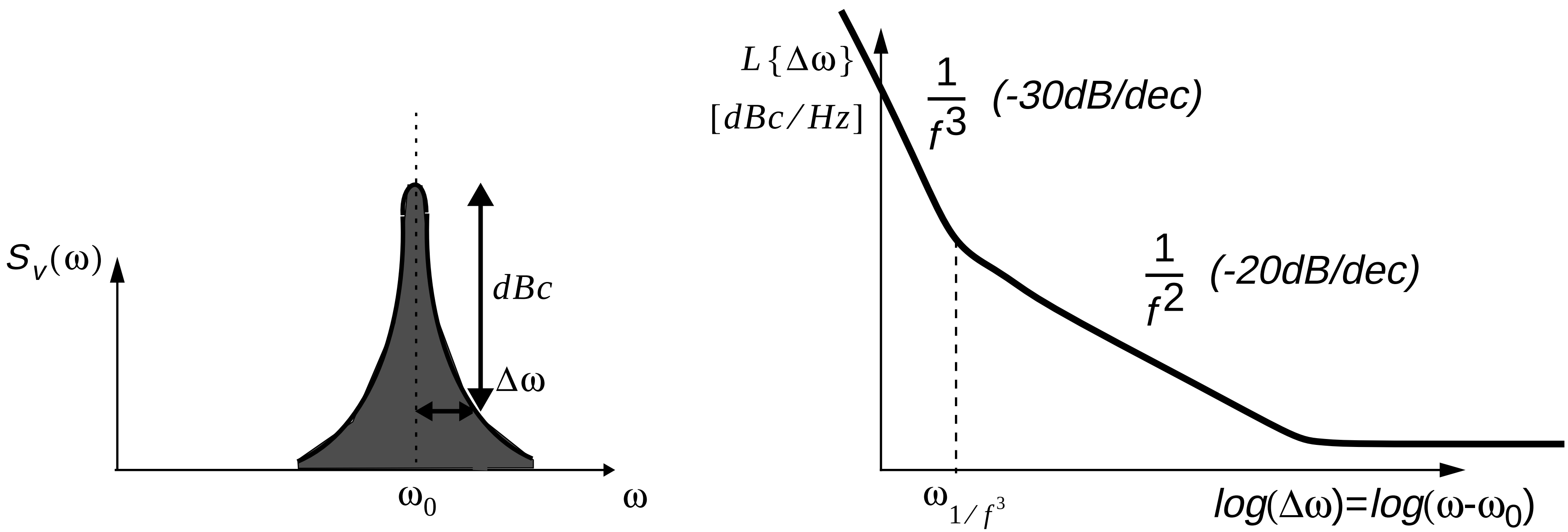
Outline

- ***Review of Time-Variant Phase Noise Model***
- Calculation of the ISF for Ring Oscillators
- Expression for Phase Noise of Ring Oscillators
- Measurement Results
- Conclusion

Phase Noise





Measured in dB below carrier per unit bandwidth.

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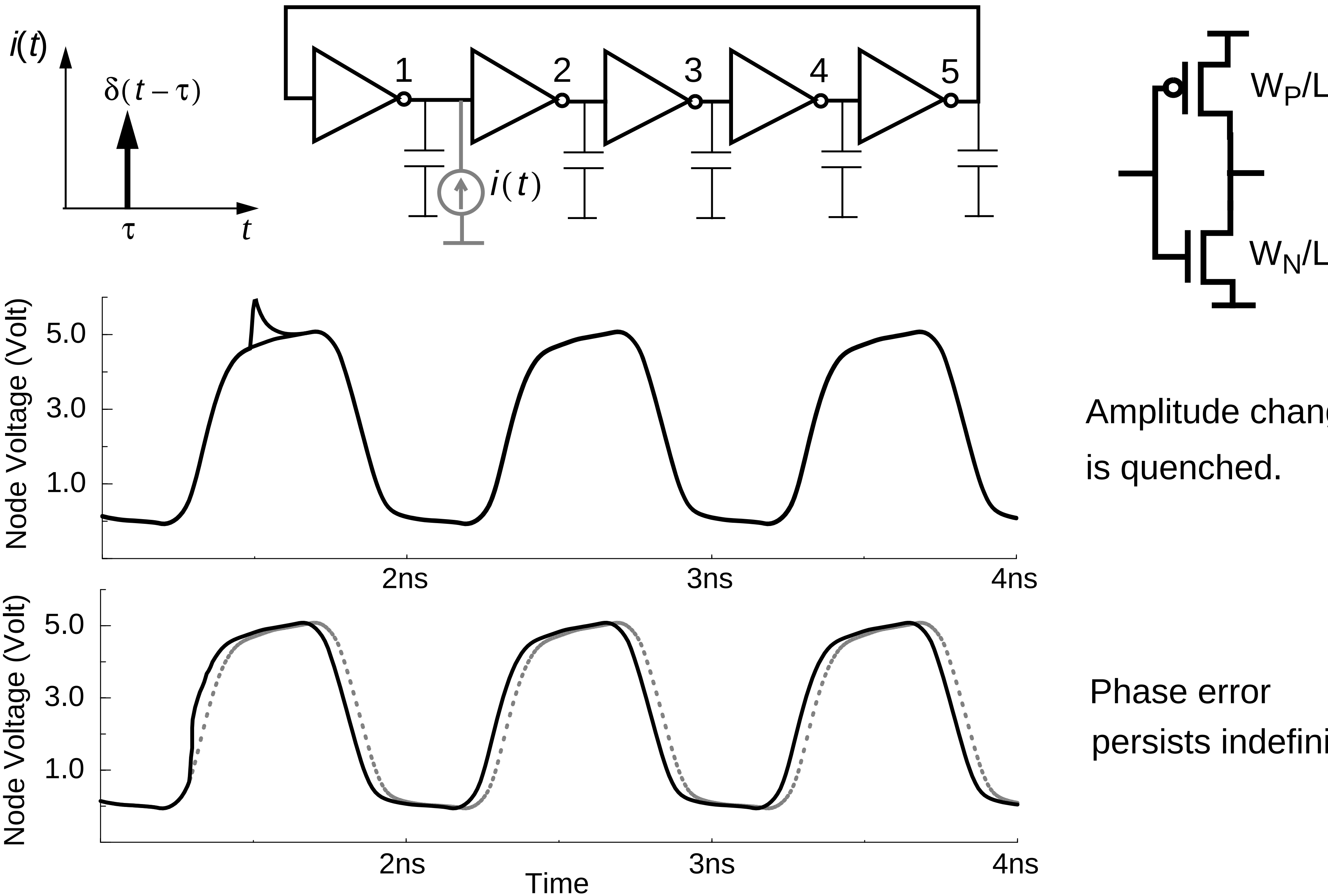
Oscillators Are Time-Variant Systems



Amplitude change is quenched.

Phase error persists indefinitely.

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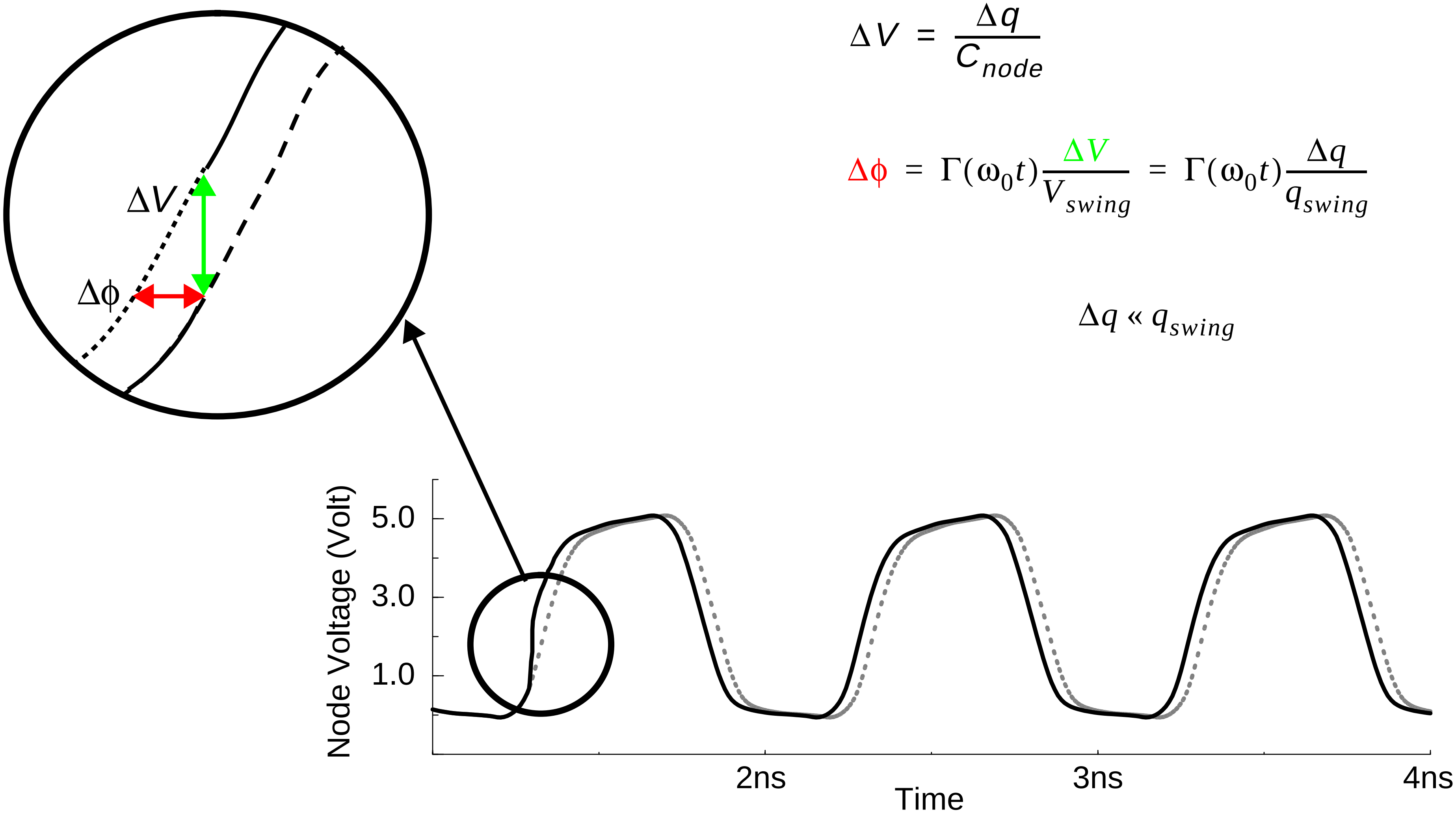
Impulse Response of a Ring Oscillator

$$\Delta V = \frac{\Delta q}{C_{node}}$$

$$\Delta\phi = \Gamma(\omega_0 t)\frac{\Delta V}{V_{swing}} = \Gamma(\omega_0 t)\frac{\Delta q}{q_{swing}}$$

$$\Delta q \ll q_{swing}$$



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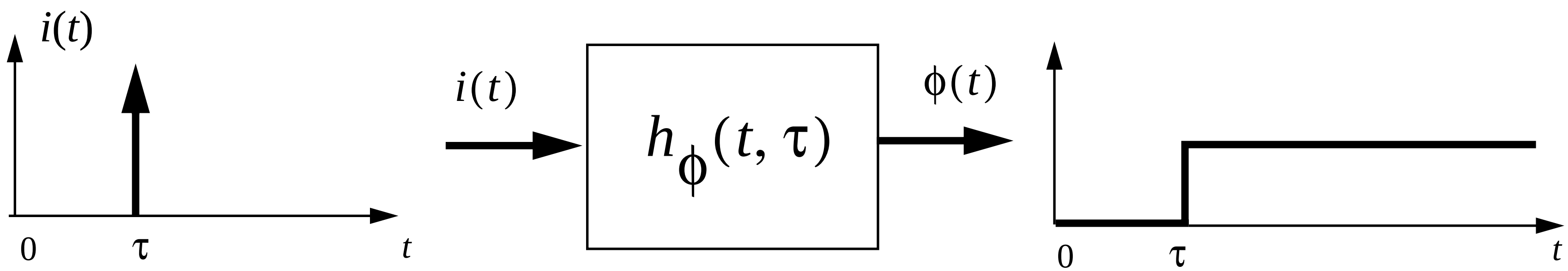
Phase Impulse Response

The phase impulse response of an arbitrary oscillator is a time varying step.



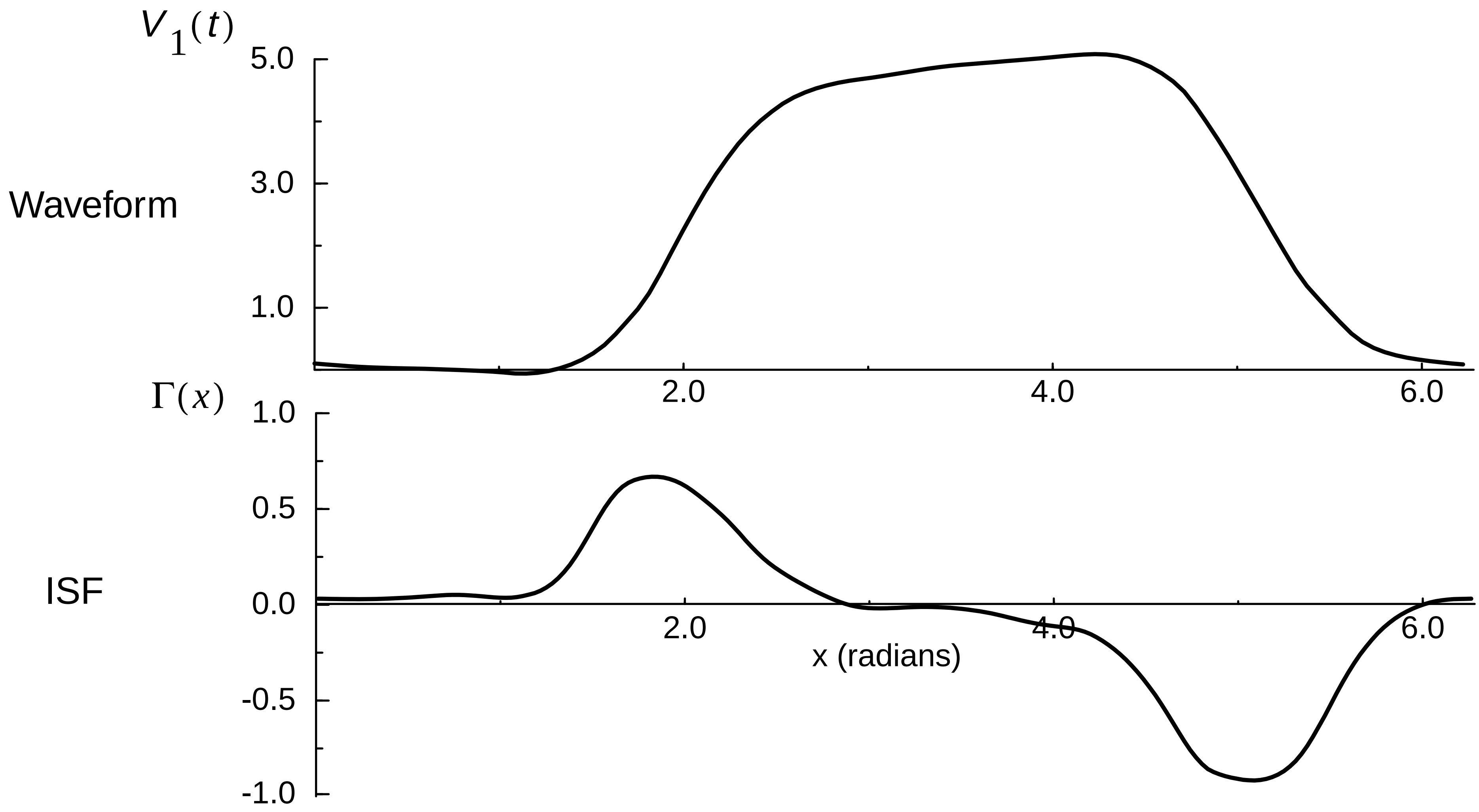
The unit impulse response is:

$$h_{\phi}(t, \tau) = \frac{\Gamma(\omega_o \tau)}{q_{max}} u(t - \tau)$$

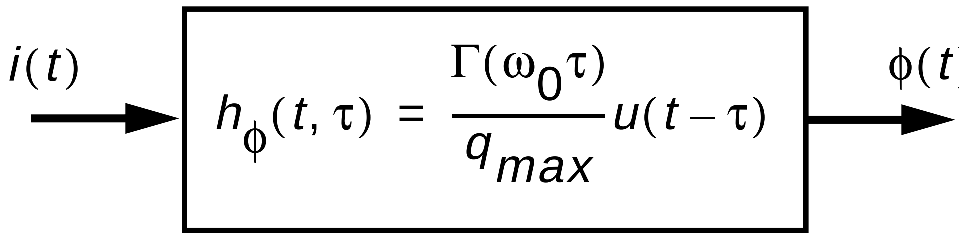
$\Gamma(x)$ is a dimensionless function periodic in 2π, describing how much phase change results from applying an impulse at time: $t = T\frac{x}{2\pi}$

Impulse Sensitivity Function (ISF)

The ISF quantifies the sensitivity of every point in the waveform to perturbations.



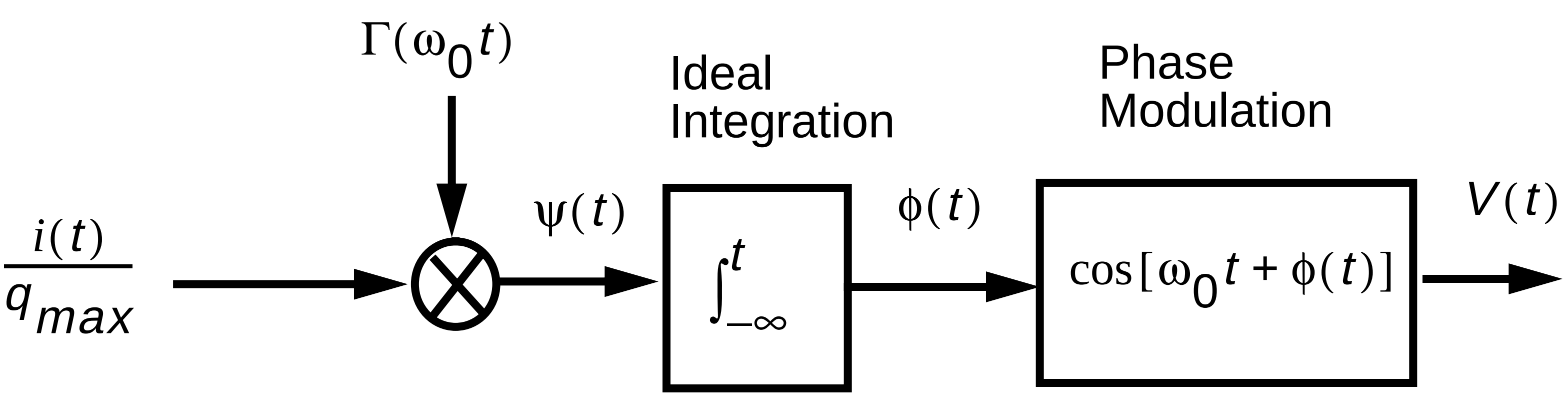
Phase Response to an Arbitrary Source



Superposition Integral:

$$\phi(t) = \int_{-\infty}^{\infty} h_\phi(t,\tau) i(\tau) d\tau = \int_{-\infty}^{t} \frac{i(\tau)}{q_{max}} \Gamma(\omega_0\tau) d\tau$$

Equivalent representation:



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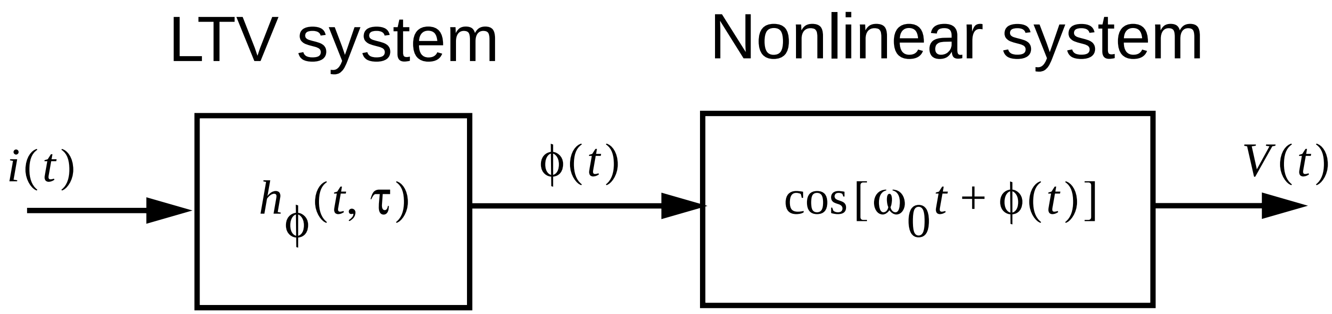
Phase Noise Due to White Noise



For a white input noise current with the spectral density of $\dfrac{\overline{i_n^2}}{\Delta f}$

The phase noise sideband power below carrier at an offset of $\Delta\omega$ *is:*

$$L\{\Delta\omega\} = N \cdot \frac{\Gamma_{rms}^2}{q_{max}^2} \cdot \frac{\overline{i_n^2}/\Delta f}{2\Delta\omega^2}$$

Γ_{rms} *is the rms value of the ISF.*

[A. Hajimiri and T.H. Lee, “A general model of phase noise in electrical oscillators,” *IEEE Journal of Solid-State Circuits,* vol. 33, no. 2, Feb. 1998.]

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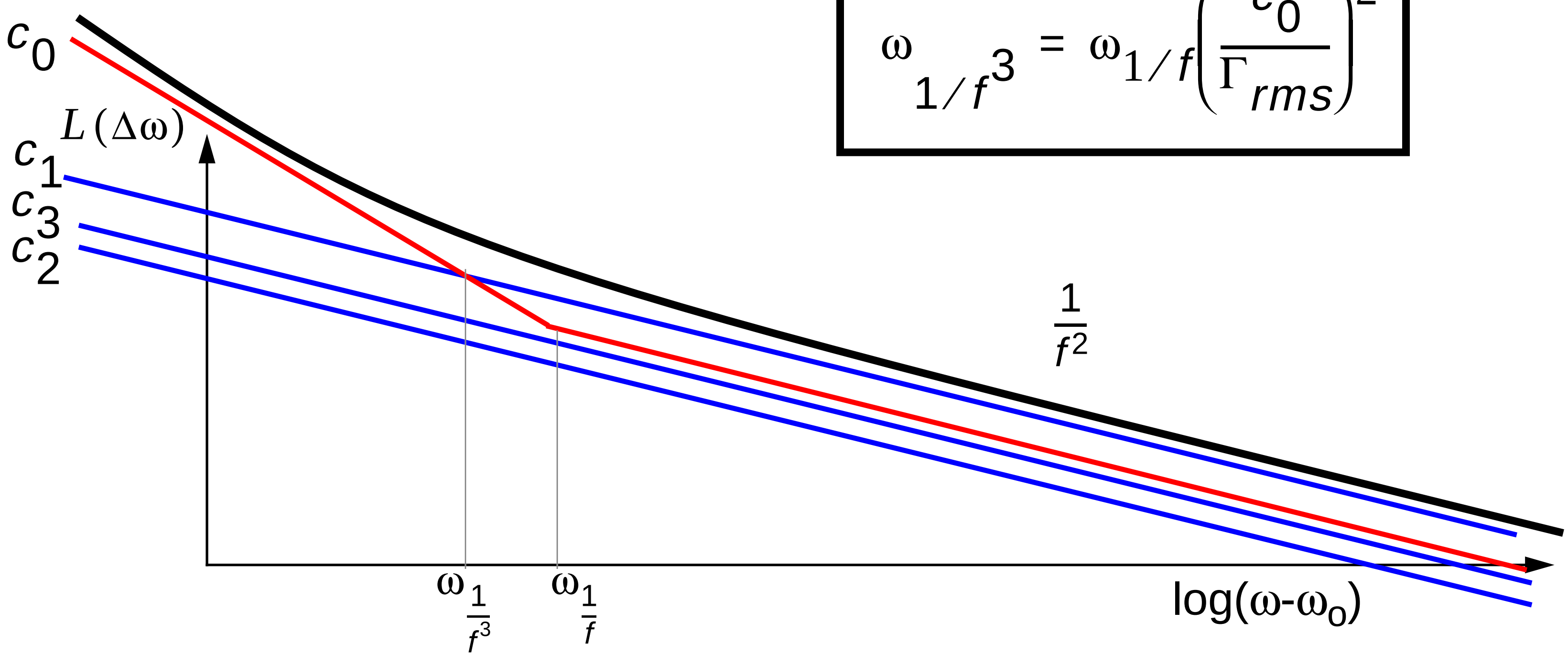
$1/f^3$ Corner of Phase Noise Spectrum

The $1/f^3$ corner of phase noise is NOT the same as 1/f corner of device noise

$$\omega_{1/f^3} = \omega_{1/f}\left(\frac{c_0}{\Gamma_{rms}}\right)^2$$



By designing for a symmetric waveform, the performance degradation due to low frequency noise can be minimized.

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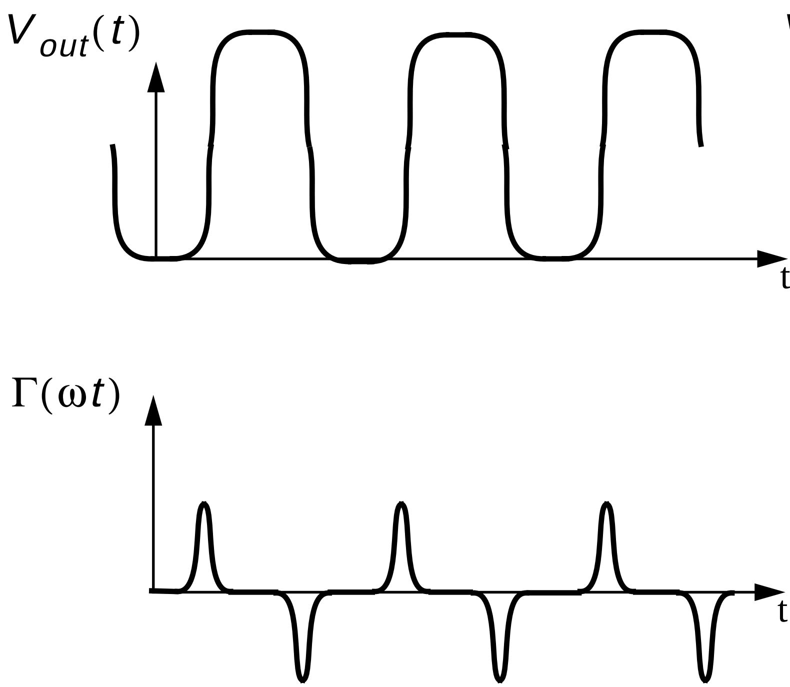
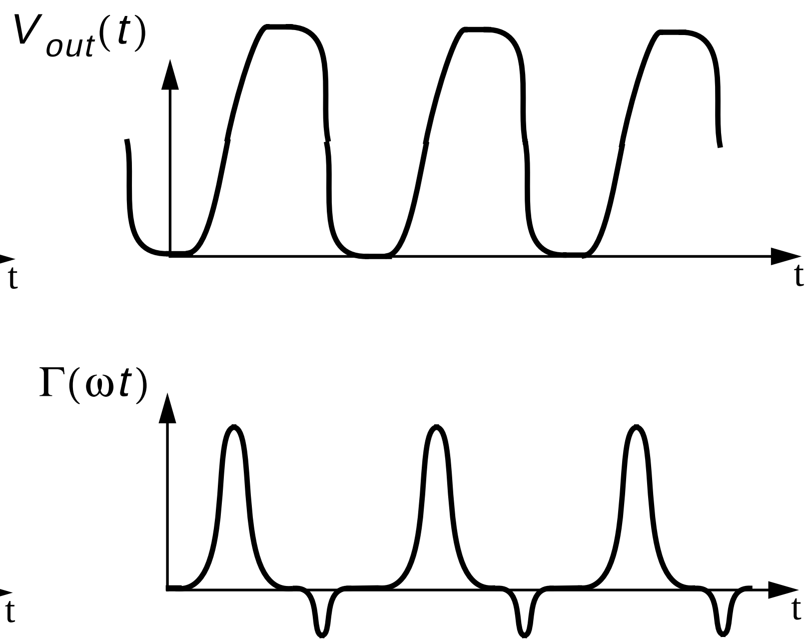
Effect of Symmetry

$$c_0 = \frac{1}{2\pi}\int_0^{2\pi}\Gamma(x)dx$$

Symmetric rise and fall time



Asymmetric rise and fall time



The dc value of the ISF is governed by rise and fall time symmetry, and controls the contribution of low frequency noise to the phase noise.

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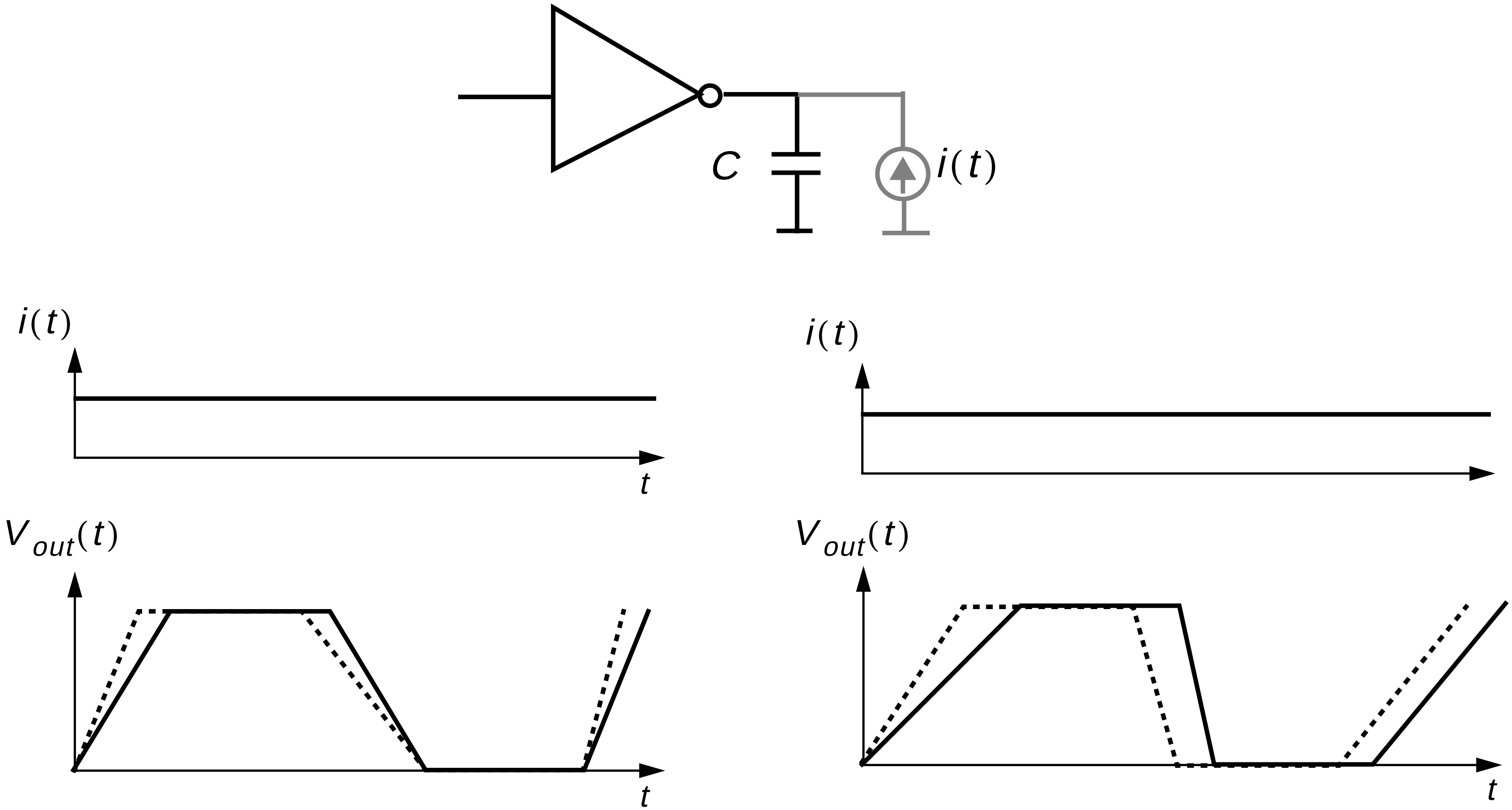
Effect of Symmetry



A low frequency current induces a frequency change for the asymmetric waveform.

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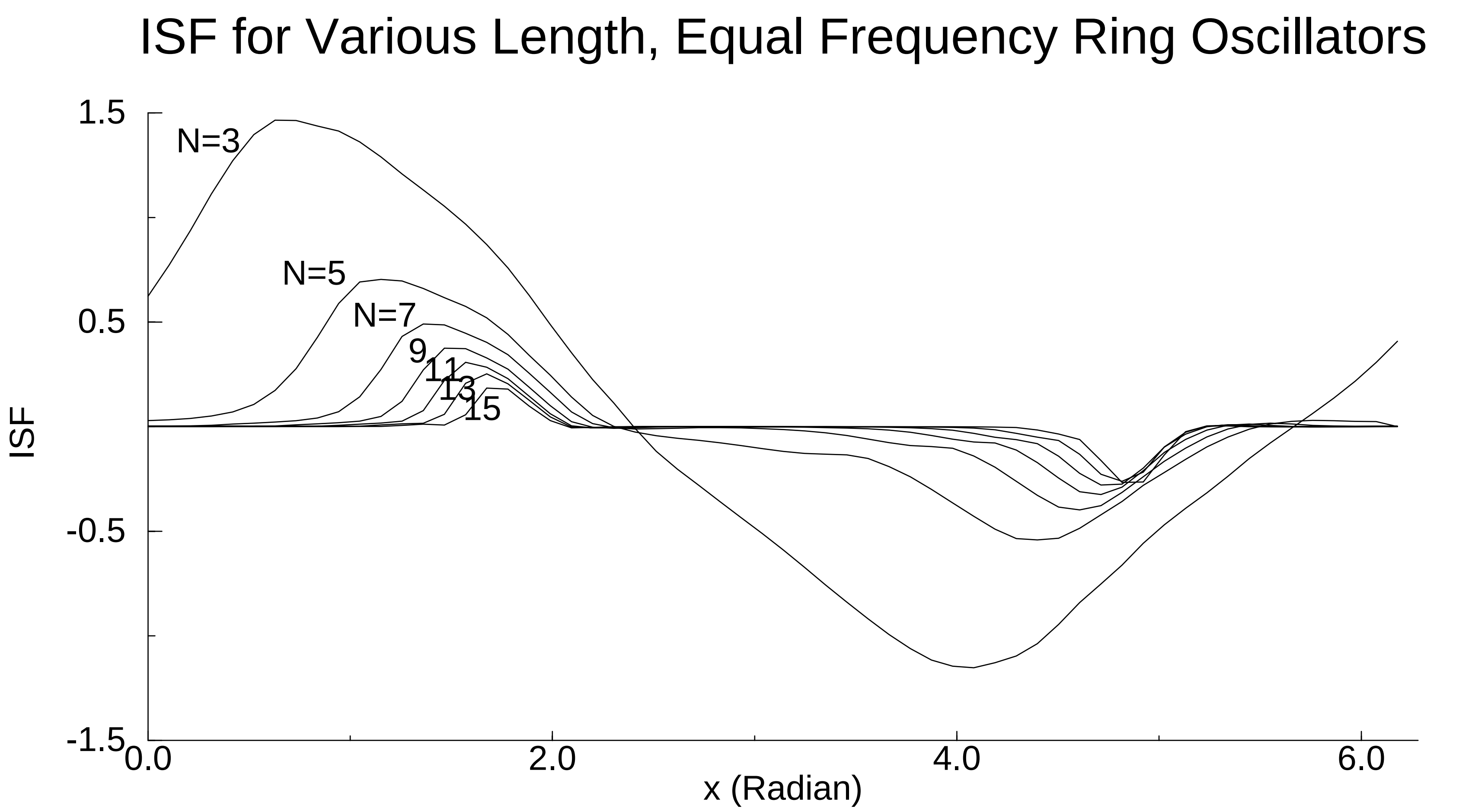
Effect of Number of Stages on the ISF



The ISF for different number of stages can be approximated by similar triangles.

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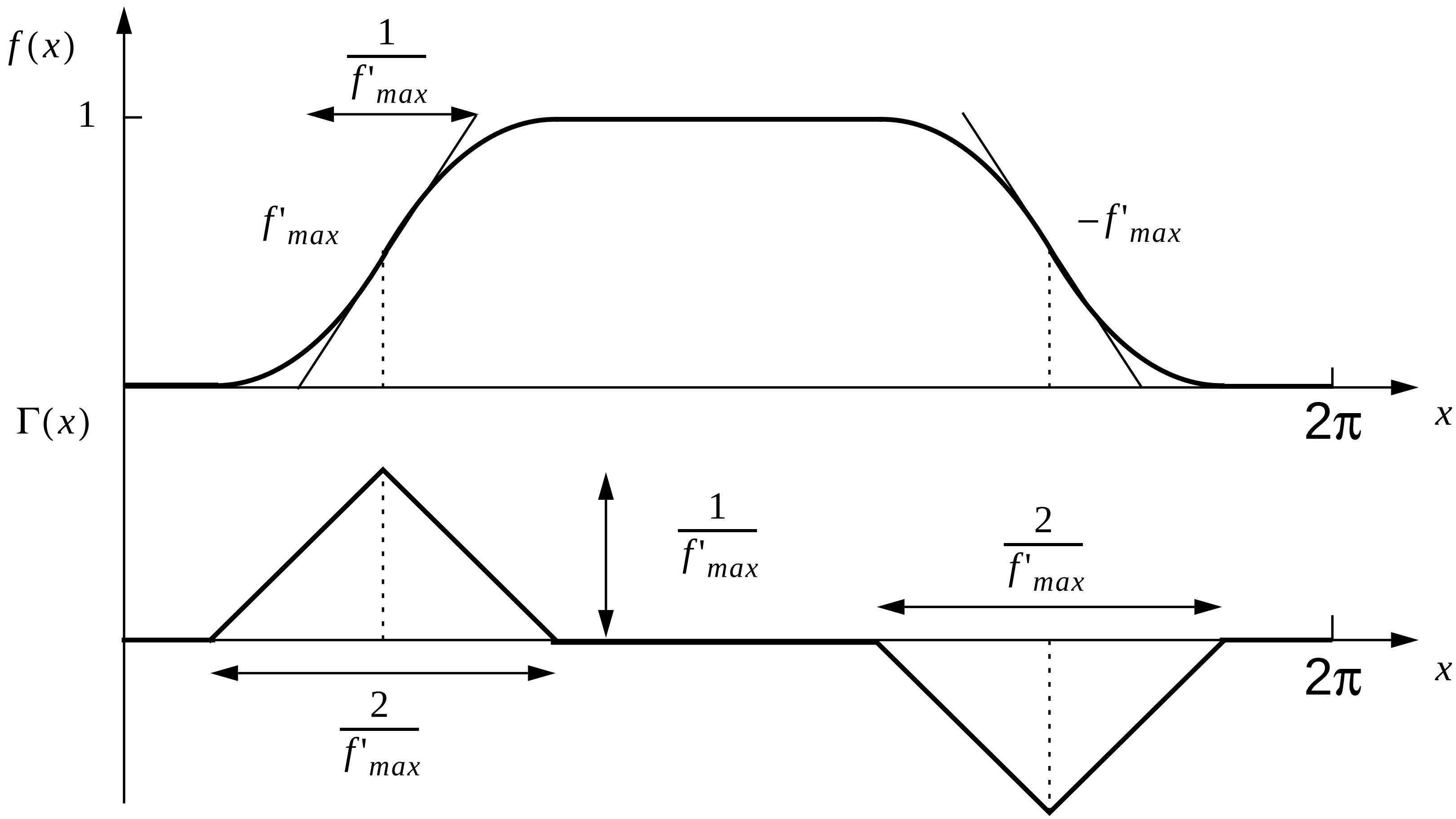
Approximate ISF for Ring Oscillators

The peak of the ISF is inversely proportional to the maximum slope of the normalized waveform.



$$\Gamma_{rms}^2 = \frac{1}{2\pi}\int_0^{2\pi}\Gamma^2(x)dx = \frac{4}{2\pi}\int_0^{1/f'_{max}} x^2 dx = \frac{2}{3\pi}\left(\frac{1}{f'_{max}}\right)^3$$

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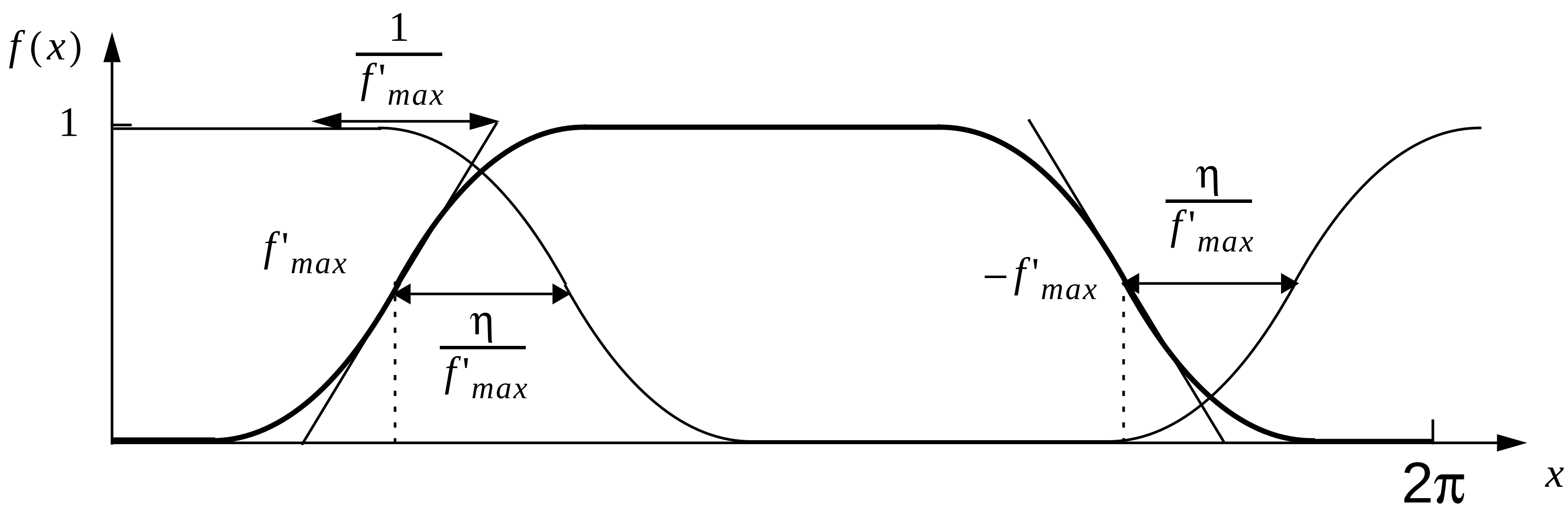
Risetime and Delay Relationship



Stage Delay: $t_D = \frac{\eta}{f'_{max}}$

Period: $2\pi = 2Nt_D = \frac{2N\eta}{f'_{max}} \Rightarrow \frac{1}{f'_{max}} = \frac{\pi}{N\eta}$

ISF RMS: $$\Gamma^2_{rms} = \frac{2\pi^2}{3\eta^3}\frac{1}{N^3}$$

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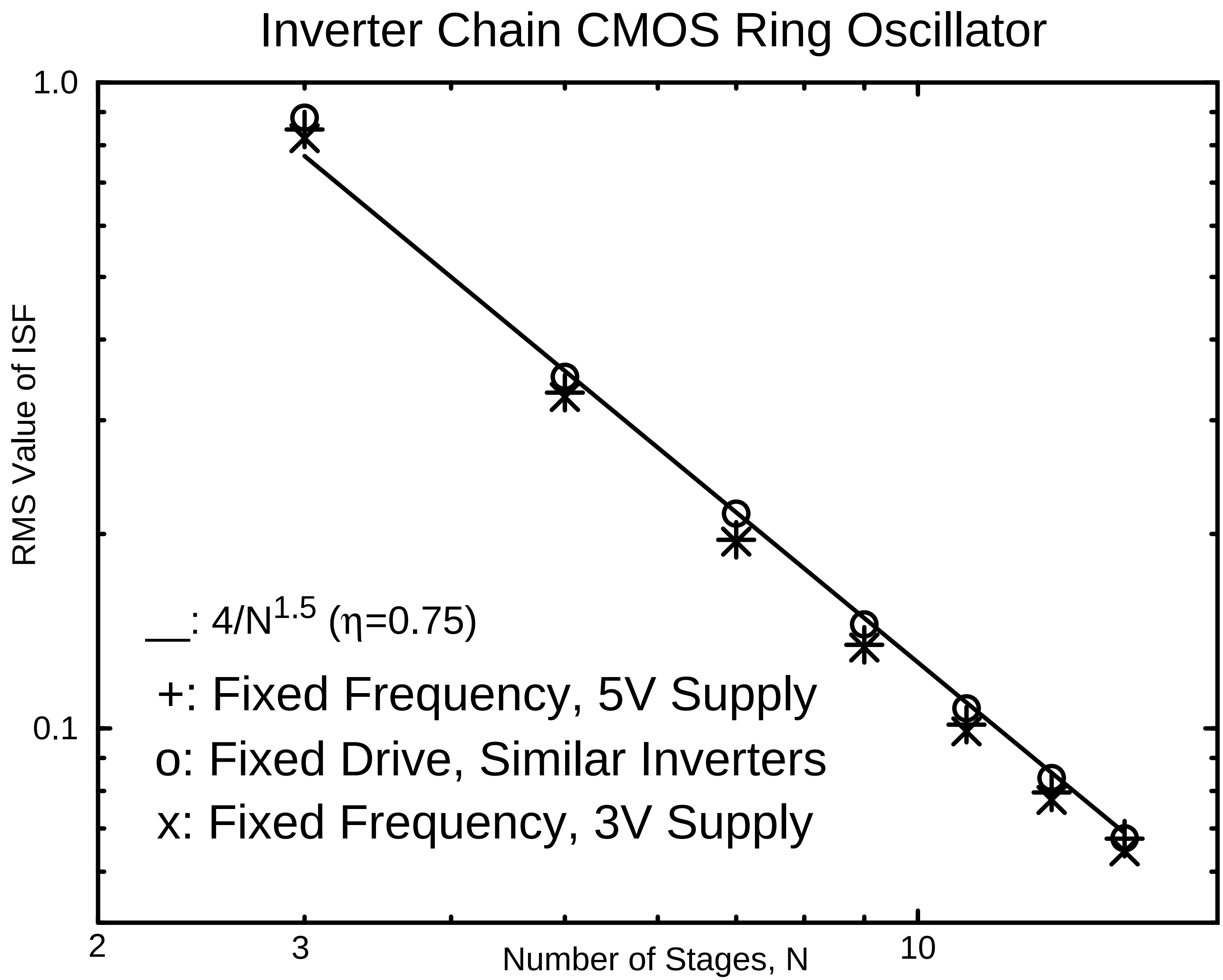
ISF RMS vs. Number of Stages



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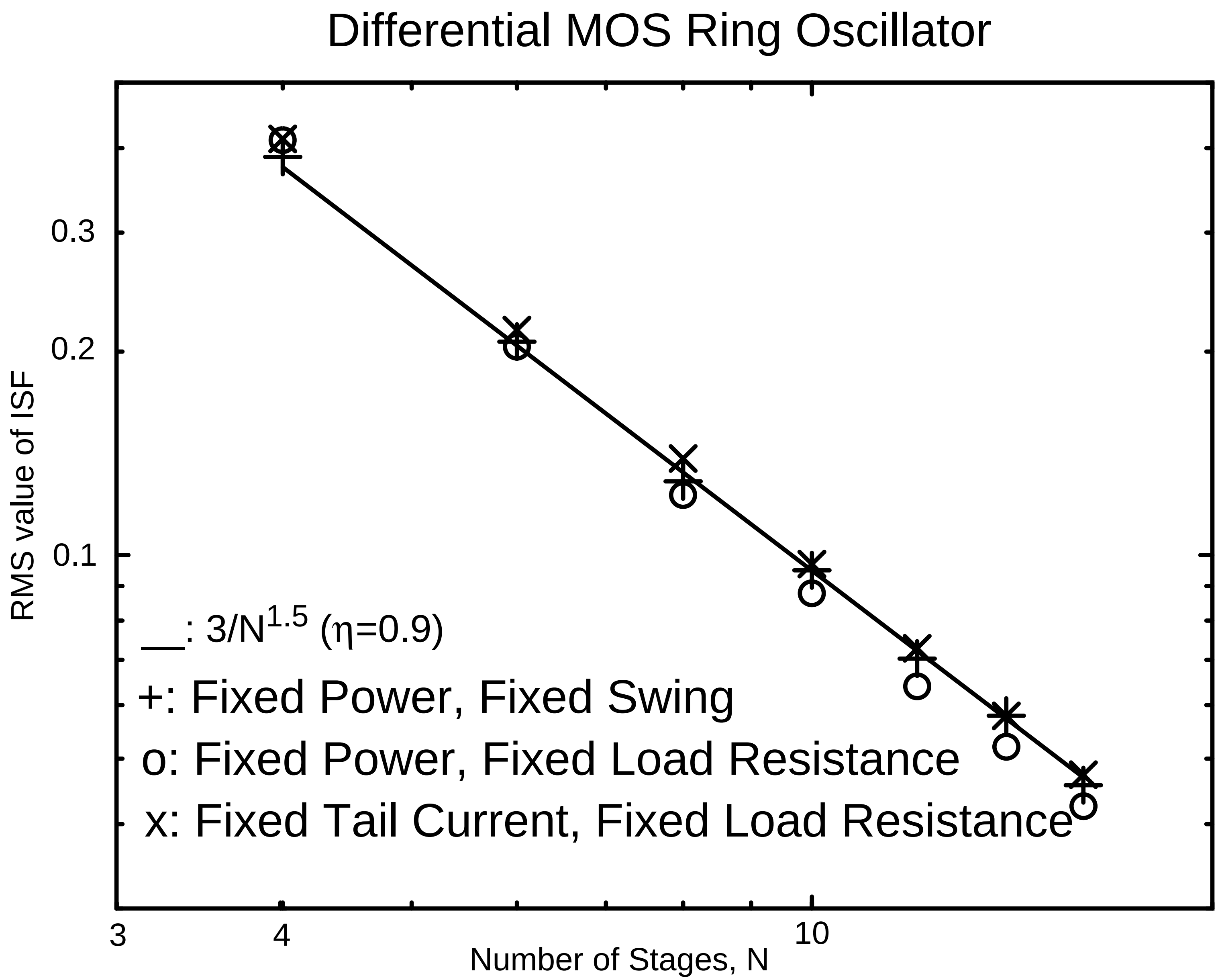
ISF RMS vs. Number of Stages



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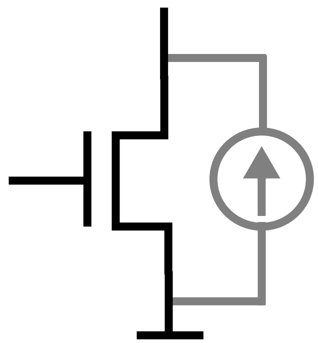
Channel Noise of MOS Transistors

$$\frac{\overline{i_n^2}}{\Delta f} = 4kT\gamma g_{d0} = 4kT\gamma\mu C_{ox}\frac{W}{L}(V_{GS} - V_T)$$



Long Channel

$$I_D = \frac{\mu C_{ox}}{2}\frac{W}{L}(V_{GS} - V_T)^2$$

$$\frac{\overline{i_n^2}}{\Delta f} = 8kT\frac{\gamma I_D}{V_{GS} - V_T}$$

$$V_{char} = (V_{GS} - V_T)/\gamma$$

Short Channel

$$I_D = \frac{\mu C_{ox}}{2}WE_c(V_{GS} - V_T)$$

$$\frac{\overline{i_n^2}}{\Delta f} = 8kT\frac{\gamma I_D}{E_C L}$$

$$V_{char} = E_c L/\gamma$$

$$\boxed{\frac{\overline{i_n^2}}{\Delta f} = 8kT\frac{I_D}{V_{char}}}$$

Valid for both short and long channel regime.

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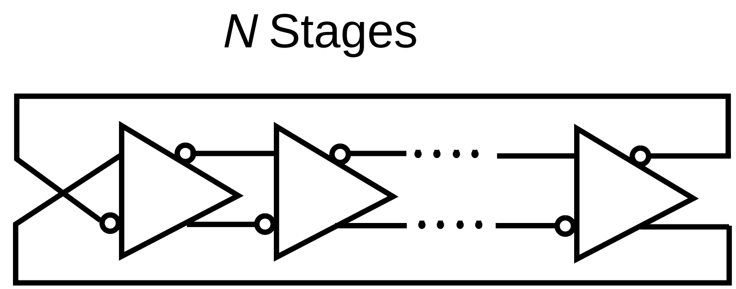
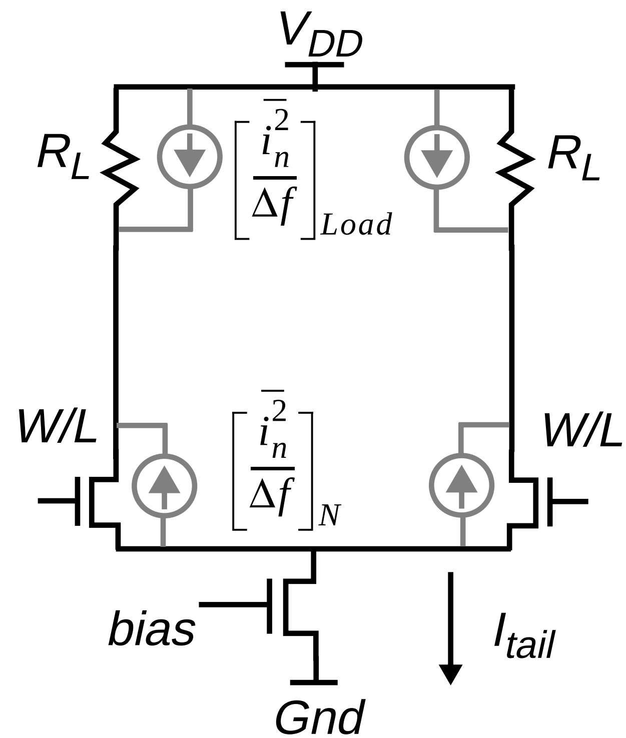
Phase Noise in Differential Ring Oscillator



Power Dissipation: $P = N I_{tail} V_{DD}$

Frequency: $f_0 = \frac{1}{2Nt_D} \approx \frac{1}{2\eta N t_r} \approx \frac{I_{tail}}{2\eta N q_{max}}$

Noise: $\frac{\overline{i_n^2}}{\Delta f} = \left(\frac{\overline{i_n^2}}{\Delta f}\right)_N + \left(\frac{\overline{i_n^2}}{\Delta f}\right)_{Load} = 4kTI_{tail}\left(\frac{1}{V_{char}} + \frac{1}{R_L I_{tail}}\right)$



$$L_{min}\{\Delta f\} \approx \frac{8}{3\eta} \cdot N \cdot \frac{kT}{P} \cdot \left(\frac{V_{DD}}{V_{char}} + \frac{V_{DD}}{R_L I_{tail}}\right) \cdot \left(\frac{f_0}{\Delta f}\right)^2$$

Short channel: $V_{char} = E_c L / \gamma$

Long channel: $V_{char} = (V_{GS} - V_T)/\gamma$

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Effect of Number of Stages on Phase Noise

For a given power and frequency, phase noise degrades with number of stages, *N*, in differential ring oscillators.

$$L_{min}\{\Delta f\} \approx \frac{8}{3\eta} \cdot N \cdot \frac{kT}{P} \cdot \left(\frac{V_{DD}}{V_{char}} + \frac{V_{DD}}{R_L I_{tail}}\right) \cdot \frac{f_0^2}{\Delta f^2}$$

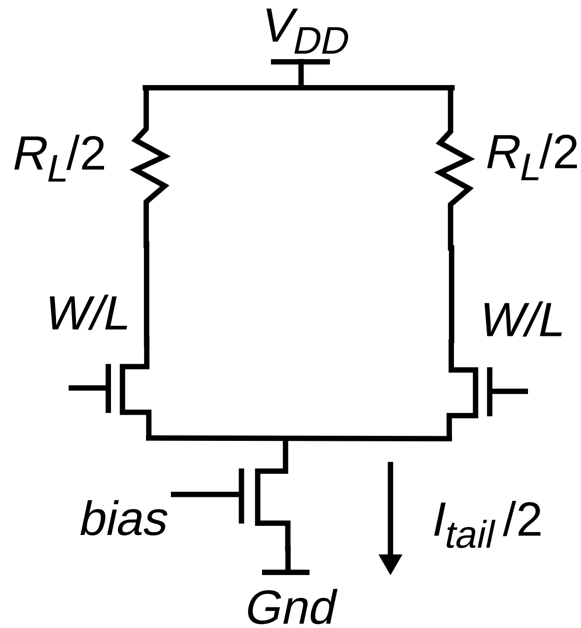
Doubling the number of stages:

R_L is divided by 2 to keep the frequency constant,

I_{tail} is divided by 2 to keep the power constant,

Therefore:

Maximum charge swing, q_{max}, is 4 times smaller.



This is ***NOT*** the case for single-ended ring, since the swing is constant.

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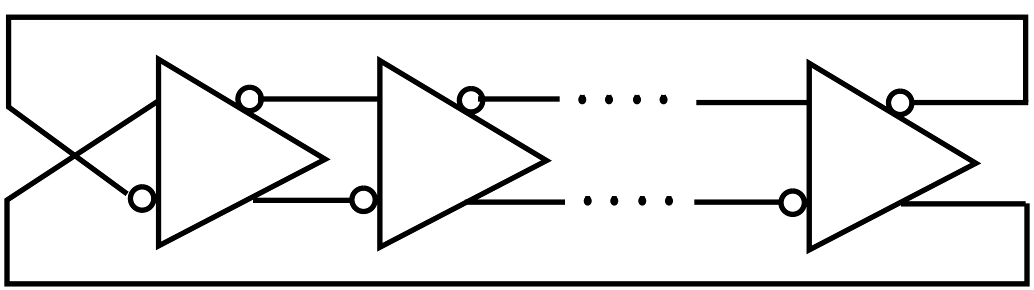
Differential Ring Oscillators

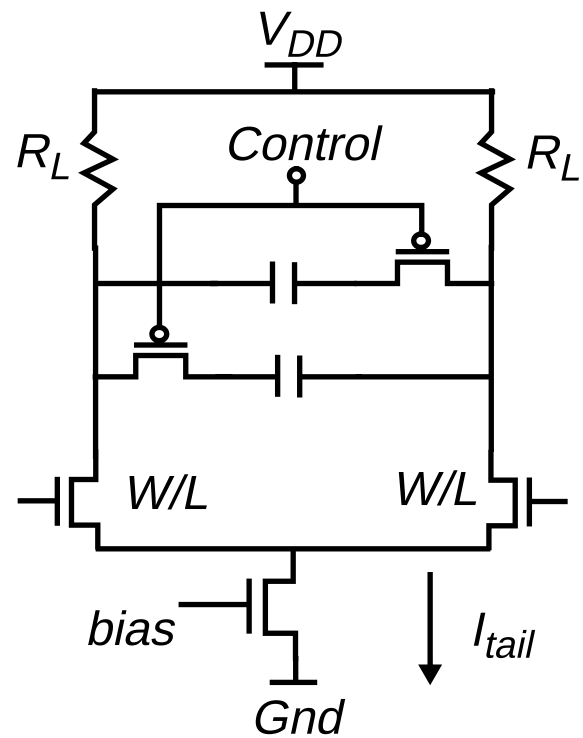
Effective channel length: L_{eff}=0.25µm.

Unsilicided poly load resistors.

Maximum frequency of 5.4GHz.

N Stages





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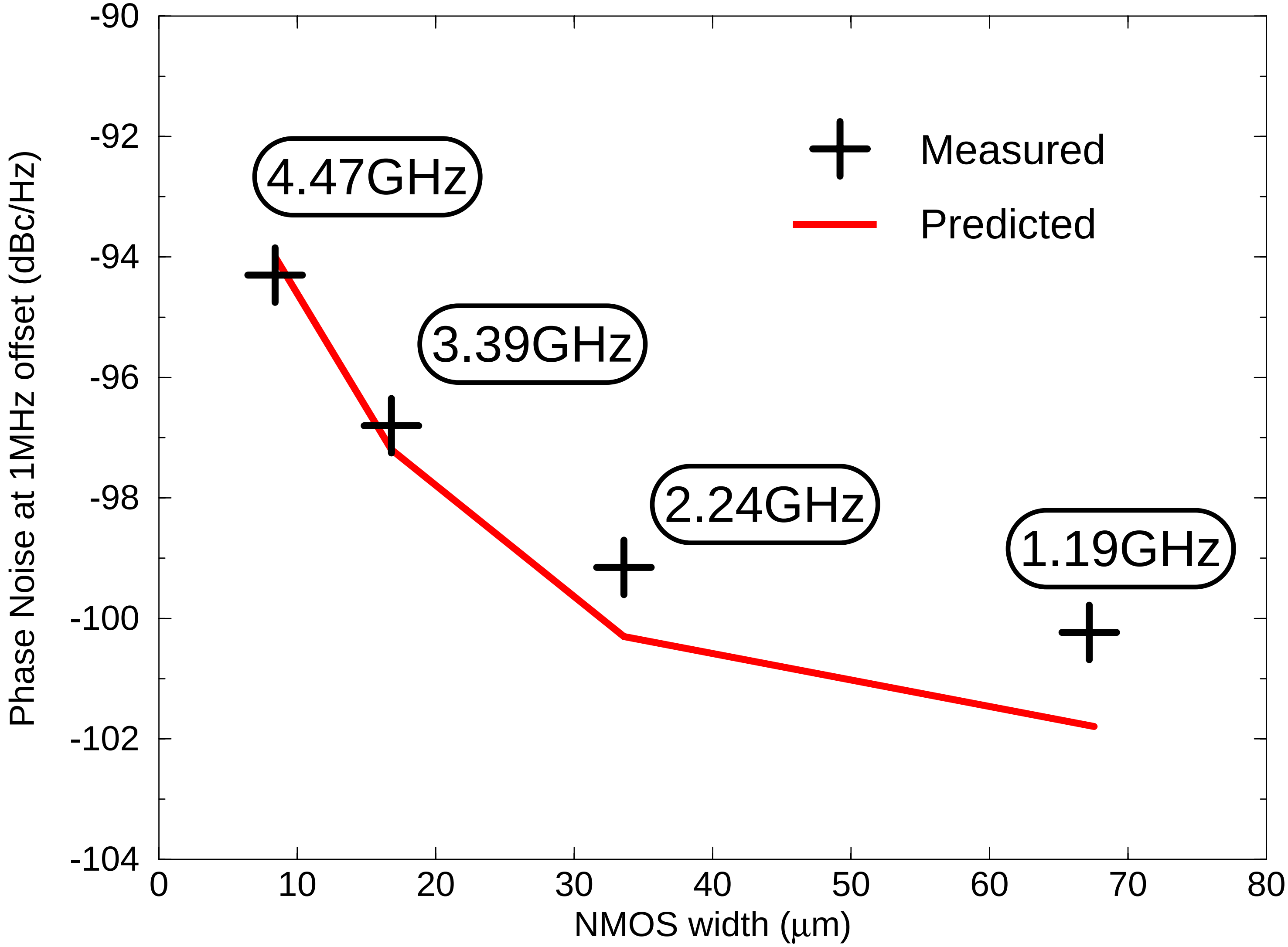
Predicted and Measured Phase Noise

Differential Ring Oscillators; R_L=1kΩ, L_{eff}=0.25μm



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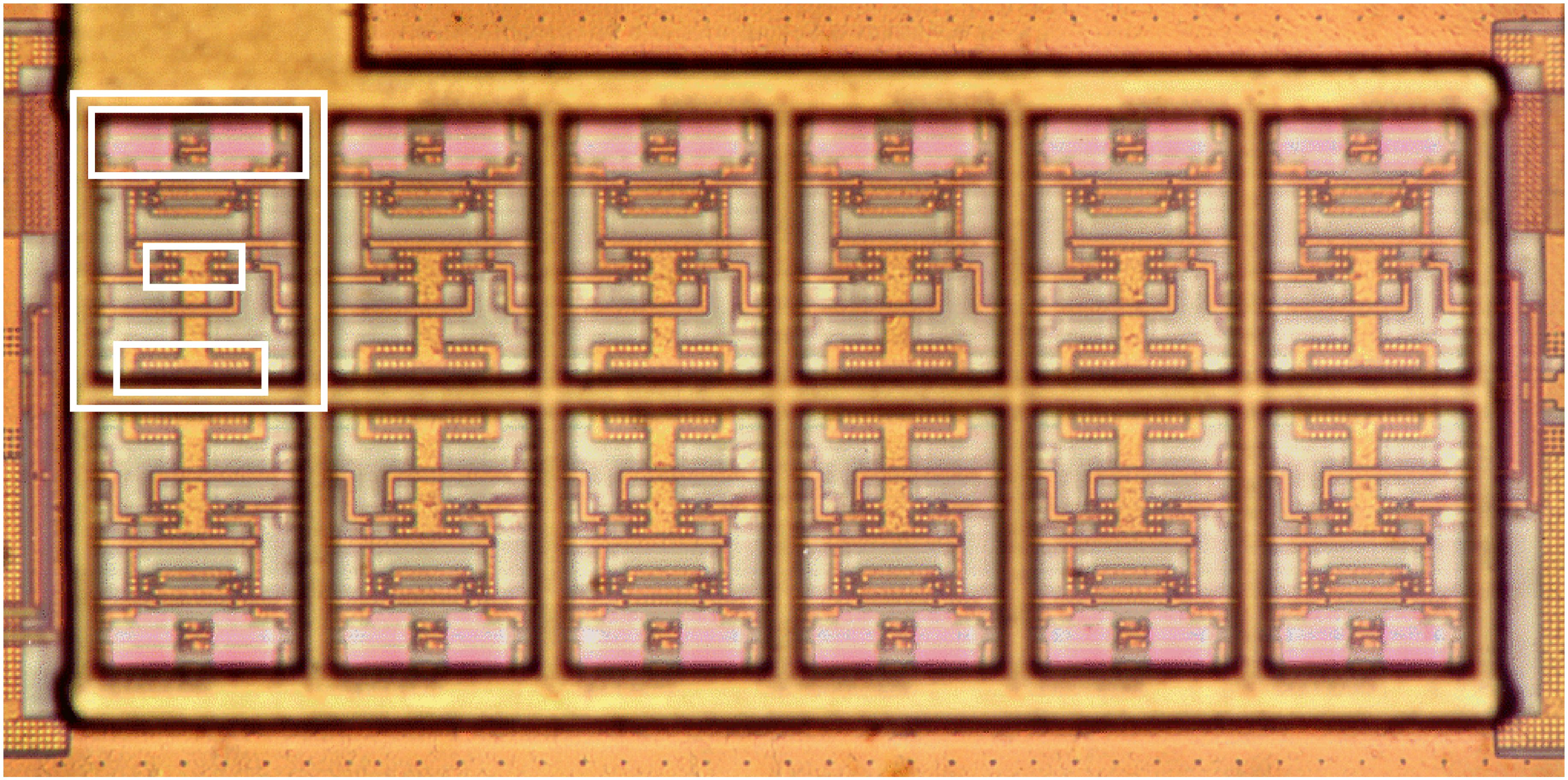
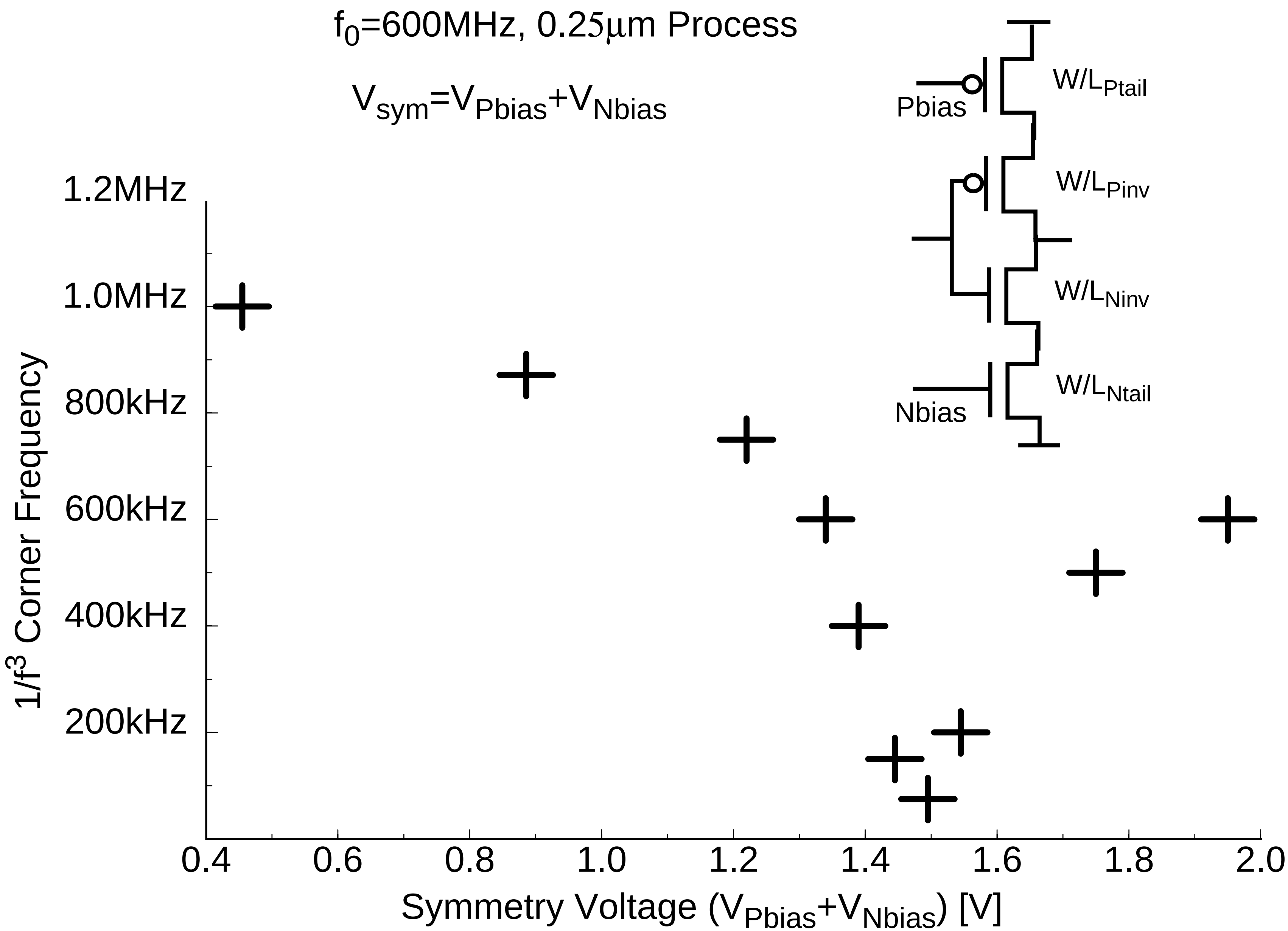
Die Photo of 12-Stage Differential Ring Osc.



9-Stage Current-Starved Single-Ended VCO



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Conclusion

- The new time-variant phase noise model is applied to ring oscillators.
- A closed form approximate expression for the ISF is obtained.
- An expression for the phase noise of ring oscillators is derived.
- The effect of number of stages on the phase noise is discussed.
- Reduction in the upconversion of 1/*f* noise due to symmetry is shown.
- Good agreement between theory and measurements is observed.